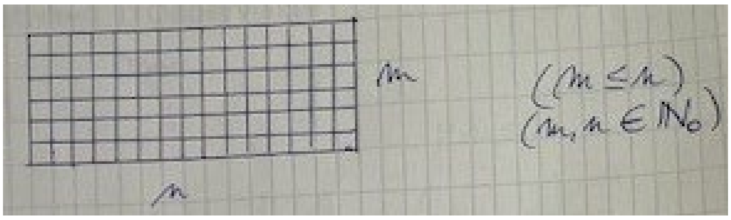
The problem:
The floor of a rectangular room is covered with (entire) square tiles. The room is m tiles wide and n tiles long. Half of the tiles are on the edge. For how many room sizes is this possible?
(A) none (B) 1 (C) 2 (D) 3 (E) more than 3
(Source: Flemish Mathematics Olympiad volume 1996, 2e round, question 18)

What is given, is shown in the drawing opposite.



Furthermore, it is given that half of the tiles lie on the side of the room. So along the length and width of the rectangle.

<u>Solution</u>:

The total number of tiles is $m.n$. So half of this is $\frac{m \cdot n}{2}$ (1)

The number of tiles lying on the side is $2m + 2n - 4$ (2). After all, the four tiles on the corner count double if we start with $2m + 2n$ to determine the tiles on the edge.

Another reasoning is that there are $2m$ tiles along the width of the room and then there are 2 times $n - 2$ tiles at the 2 lengths. The expression then becomes : $2m + 2(n - 2) = 2m + 2n - 4$.

If this must be equal to half the total number of tiles, then the equality must be true ((2)=(1)) :

$$2m + 2n - 4 = \frac{m \cdot n}{2}$$

We express m as a function of n but, of course, this can also be done in reverse.

$$2m + 2n - 4 = \frac{m \cdot n}{2} \Leftrightarrow 4m + 4n - 8 = m \cdot n \Leftrightarrow 4m - m \cdot n = 8 - 4n$$

$$\Leftrightarrow m(4 - n) = 8 - 4n \Leftrightarrow m = \frac{8 - 4n}{4 - n}$$

We know that $m \leq n$ and that both numbers are natural numbers but not equal to 0. Since the denominator is $4 - n$ is, we have: $4 - n \neq 0 \Leftrightarrow n \neq 4$

Natural numbers are discrete, allowing us to examine 'all' of them in a table. We choose natural values for n and calculate the corresponding value for m

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	…	$+\infty$
m	$\frac{4}{3}$	0	-12	/	12	8	$\frac{8}{3}$	6	$\frac{28}{5}$	$\frac{16}{3}$	$\frac{36}{7}$	5	$\frac{44}{9}$	$\frac{24}{5}$		4

We notice that only a few natural values occur. Furthermore, we see that the following numbers we calculate for m at $n > 12$ no longer yield a natural number. We can confirm this with the limit: $\lim\limits_{n \to +\infty} \frac{8-4n}{4-n} = \lim\limits_{n \to +\infty} 4 = 4$. The graph provides additional information about the real function and confirms that we can conclude from the table that only 2 situations are possible.

Answer: option C is the correct answer.