ProTEEM

TEAM UP FOR PROTEEM

Co-funded by the European Union

Module 2: Didactics in Mathematics – Day 4 Part I

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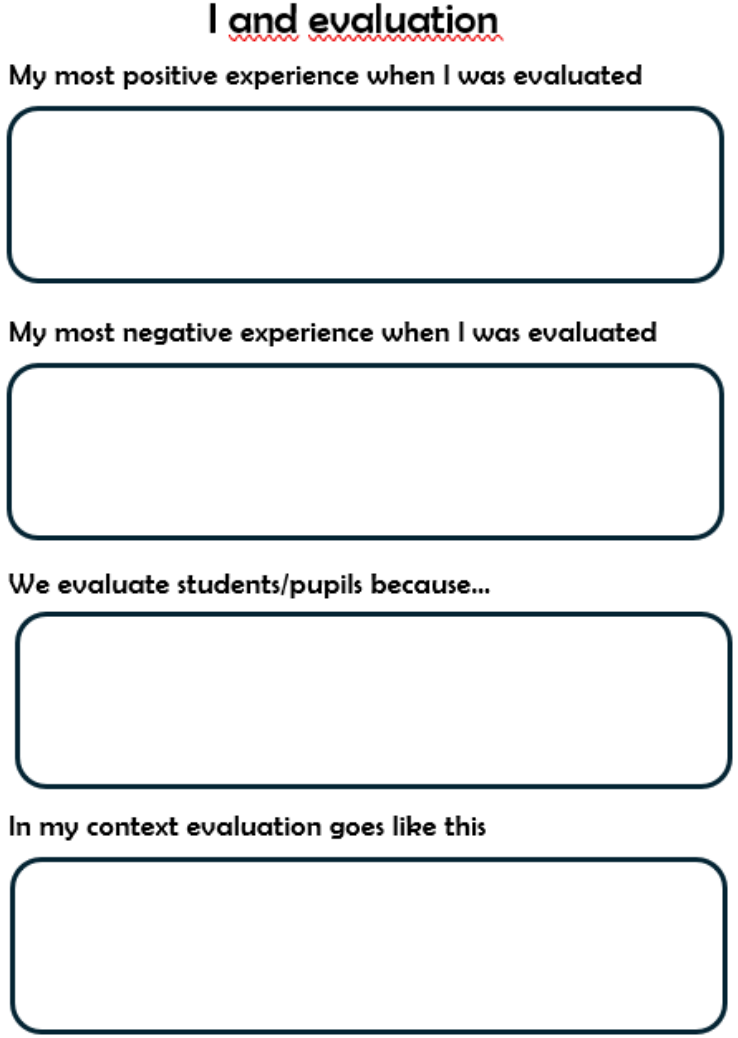
Mrs. Gilberte Verbeeck

Universiteit Antwerpen
ASOE | Antwerp School of Education

Evaluation: you when you were pupil – your context

Complete the worksheet 10’



I and evaluation

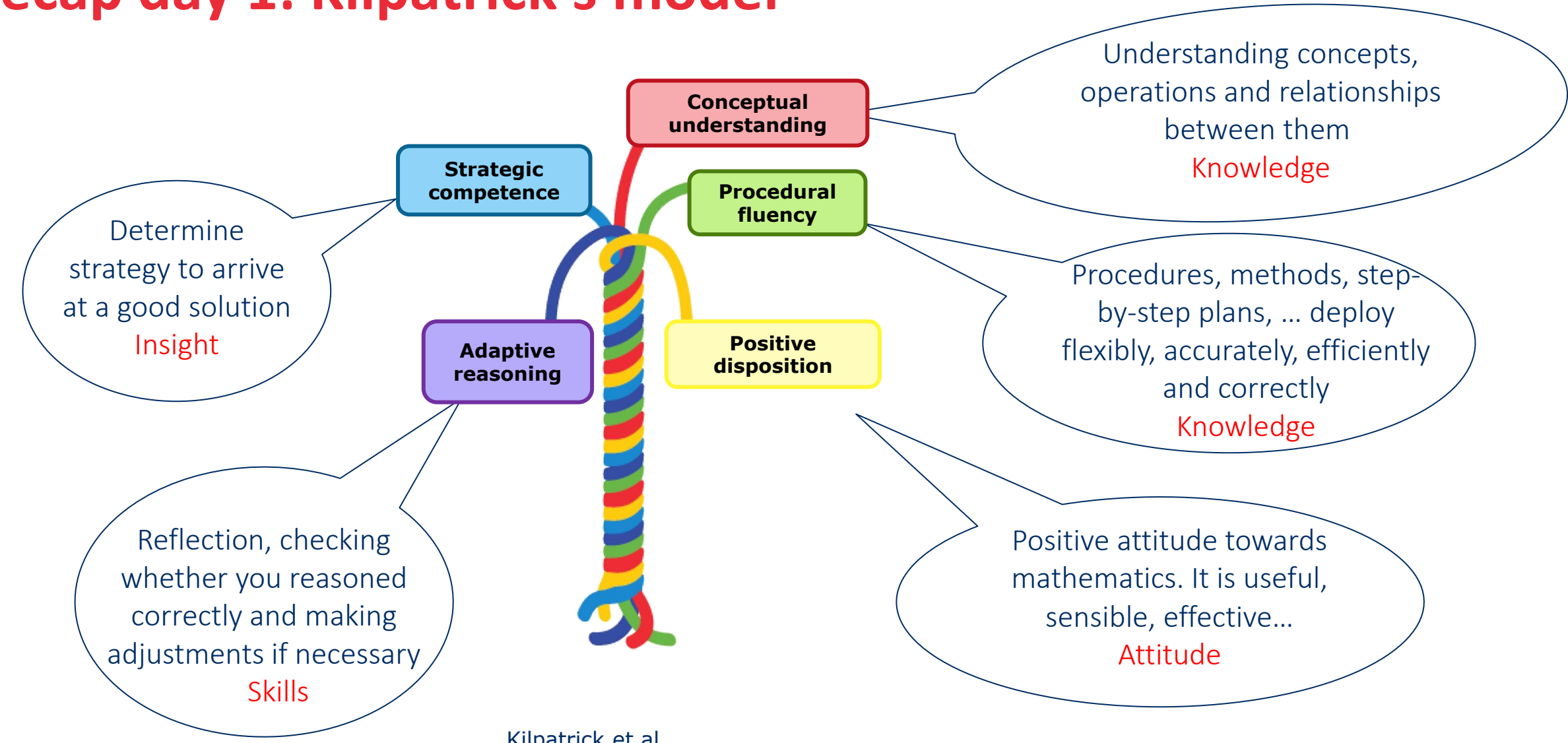
My most positive experience when I was evaluated

My most negative experience when I was evaluated

We evaluate students/pupils because...

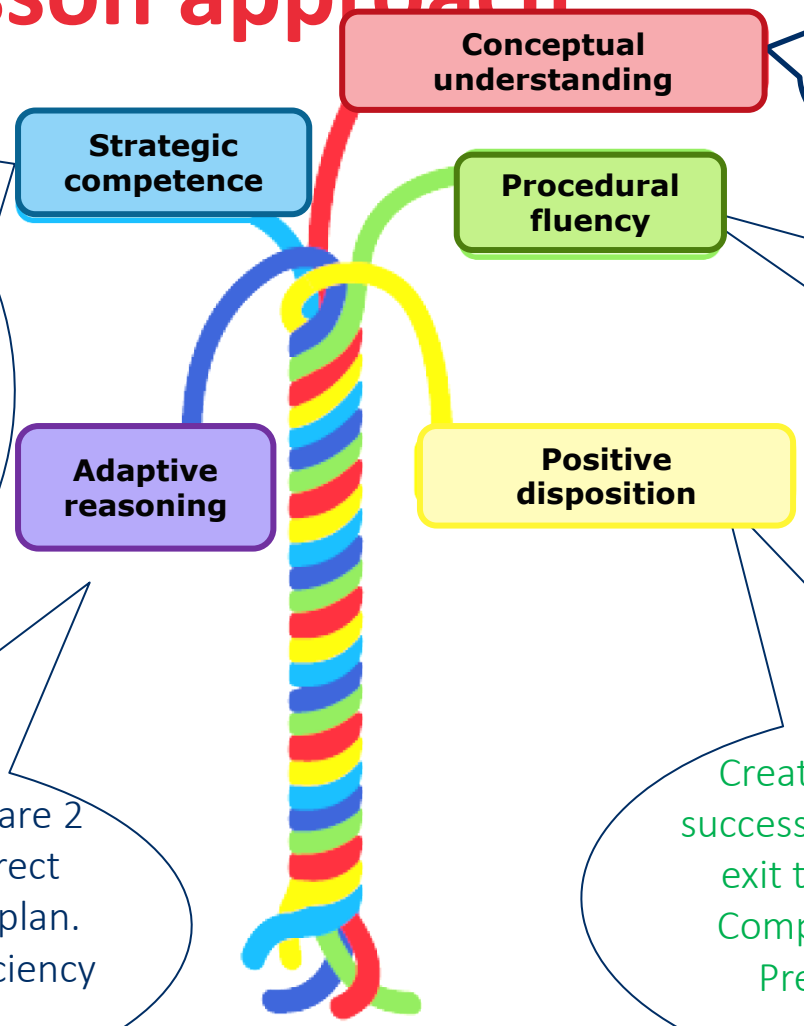
In my context evaluation goes like this

Recap day 1: Kilpatrick's model



Kilpatrick et al.

Objective - > lesson approach



Ensuring a rich cognitive schema. Apply concepts with attention to understanding, definition and image of a concept.

Learning heuristics: What do you recognize? Can we divide the problem into partial problems? Make an outline or diagram.
Attention to different methods and efficient methods.
Attention to repetition. Attention to students' ways of thinking. Activating students.
Getting students to solve or try an exercise on their own. Discussing this

Teaching and training procedures with understanding. Making different types of exercises and applications. Analyze (parts of) solutions. Learning students' ways of thinking. Repeating

Analyze the method. Compare 2 solutions. Find errors, correct errors. Give a step-by-step plan. Explain steps. Check for efficiency and no detours.

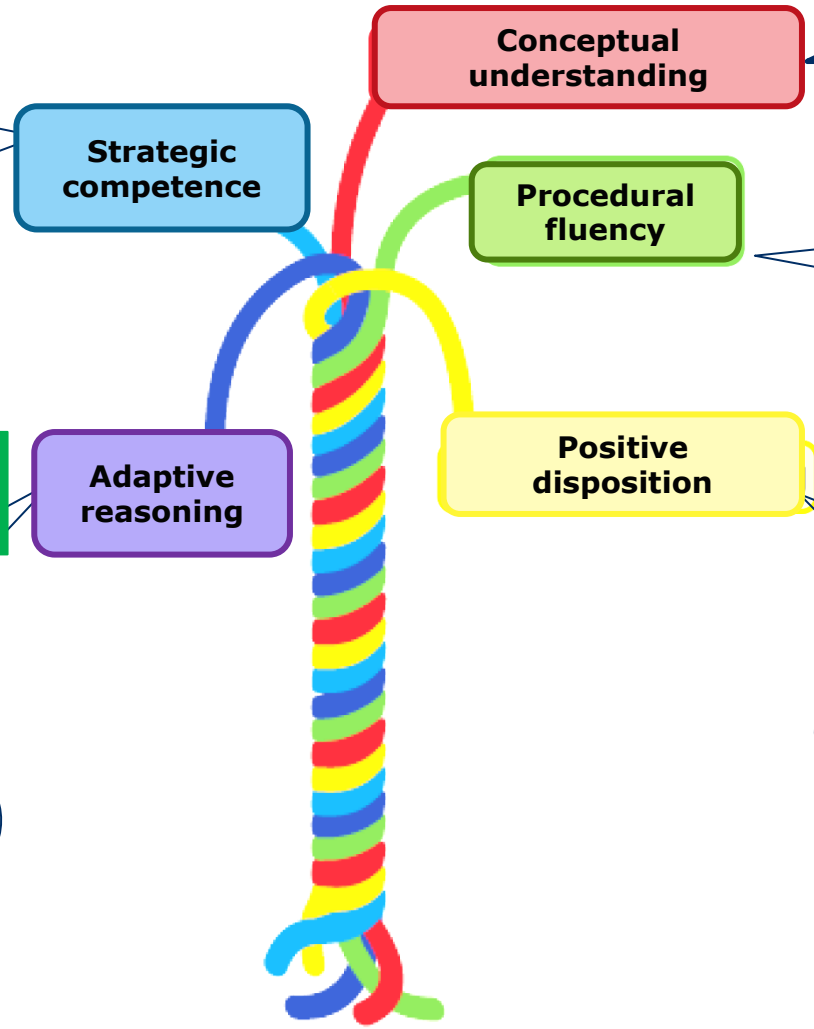
Creating opportunities for success experiences. Intro and exit tickets. Feed forward. Compliment what is good. Preparing student for evaluation...

Objective - > lesson approach - > Evaluation

Solve: $r^2 + r = 12$. What is the similarity between: 1) $sin^2x + sinx = 12$ **AND** 2) Find algebraically the intersection points of $y = x^2 + x$ en y=12.
Solve using the most efficient method. Give a plan of action. What are possible first steps that could lead to a solution? Explain this step.

Evaluating considering all strands of Kilpatrick requires other questions

Compare the following two solutions. Which one is correct/wrong/efficient? Correct errors.



Which signtable, graph and zero points belong together. Explain an aspect from definition. …

Determine discriminant. Solve the quadratic equation with sum and product rule. Create signtable.

Creating feasible tests.Formative evaluation. Allow choice - differentiate. Complimenting what is good.

Lesson approach in line with evaluation

- **Pursue that after class every student**
  - knows new concepts (KNOWLEDGE)
  - has INSIGHT of new concepts => right place in cognitive schema
- **How.**

  Ask many questions during the lesson and vary in types of questions (KilPatrick)
  - Math and standard procedures,
  - Research and modeling,
  - Explaining
  - Activating (recap day 3)
  - ....

Lesson approach and lesson plan (Recap day 3)

- What? Prestructured conversation– question & answer – goal = insight
- Pitfall? Dropping out + pretend attention + mindless copying + …
- When activating and deep learning? Good questions + questioning techniques + dealing with answers
- Importance of lesson preparation:
  - **Clear Thinking** Questions that learners **can** answer
  - **Crucial** questions for steps/concepts that **learners are struggling with**
  - **Two-step model** questions with **possible** answers
  - Dealing with non-response

Activating ← Deep learning

Non - Activating → Surface learning

Lesson approach and lessonplan: anticipate

- **What possible answers can a learner give?**

$$\frac{a^2\sqrt[4]{a}}{\sqrt{a}}$$

$$x^2 - 36 = 0$$

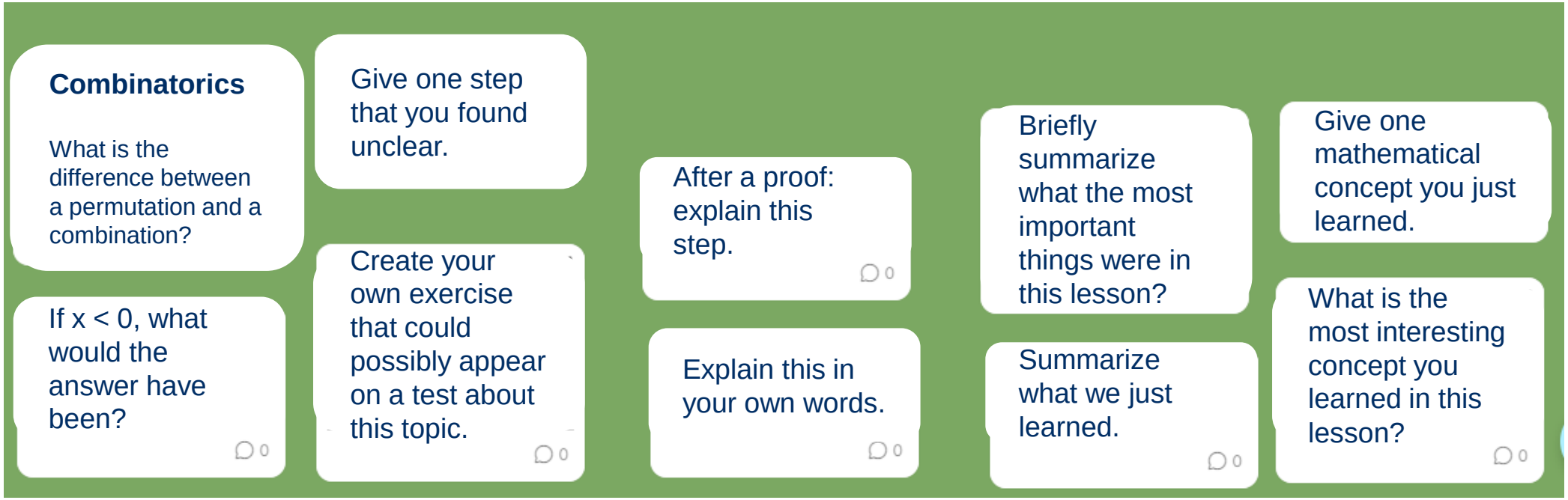
Good examples on padlet!

Lesson approach and lessonplan: anticipate

- **Good question after each finished lesson(sequence):**

  NOT: Did you understand? Are there any questions?

  Technique in lesson via padlet (day 4 - part II): padlet.com/gilberte_verbeeck/5c4a9riaqkjzrwx4



Lesson approach during lesson: coach learners

- **Further Questioning**
  - Asking for an explanation of a certain answer
  - Asking for explanation of method
- **Allowing other students to evaluate an answer**
- **Passing questions from students to other students**
- **Attention to building cognitive schemas (recap day 1)**
- **ALSO ask thinking questions, method questions, crucial questions to make the learning process effective ...**
- **Guidance group work: Avoid lecturing, focus on mini Q&A**

Lesson approach during lesson: Repeat

- **Brainstorm (via padlet.com - Blackboard – paper) (recap day 1)**

- **End of lesson sequence: ask a good question (recap lesson plan slide 9)**
- **Q&A where all learners all actively involved (recap day 3)**
- **Intro- exittickets – question box (recap day 1 - next slides)**

Lesson approach: begin or end of lesson

- **Intro – ticket**
  - Question about previous lesson
  - Orientation question to new lesson
- **Exit ticket:**
  - General Examples:
    - Make a question about the lesson, put it on one side of a paper. Write the answer on the other side of the paper.
    - Ask a quesion about the content of this lesson, that you might expect on a test.
    - How to start solving of this exercise …… ?
  - Pitfall if you prepare specific questions in your lessonplan: NOT about the last part of your lessonplan (timing – plan B!)

Intro – Exit Specific Examples

$F(x) = x^3$ is een primitieve functie van $f(x) = 3x^2$ omdat _____

De afgeleide van $F(x) = x^4 + 3$ is _____

dus een primitieve functie van $f(x) =$ _____, is _____

$\int_a^b dx = \int_a^b 1\, dx =$ _____

Een primitieve functie van $f(x) = x^4$, is $F(x) = \frac{x^5}{5} - 8$ omdat _____

Intro -> Exit: Learners receive a (paper with) question upon when entering the class room. They have to answer it att the end of the lesson.

Success experience is motivating!
Learners who know subject matter at end of lesson!

Intro – Exit examples

Review of exercise:
error in solution

Find the mistake in the answer below

Kies het juiste antwoord.

$\int_{-2}^{3} f(x)\,dx =$

A $-\frac{13}{2}$

B $-\frac{5}{2}$

C 0

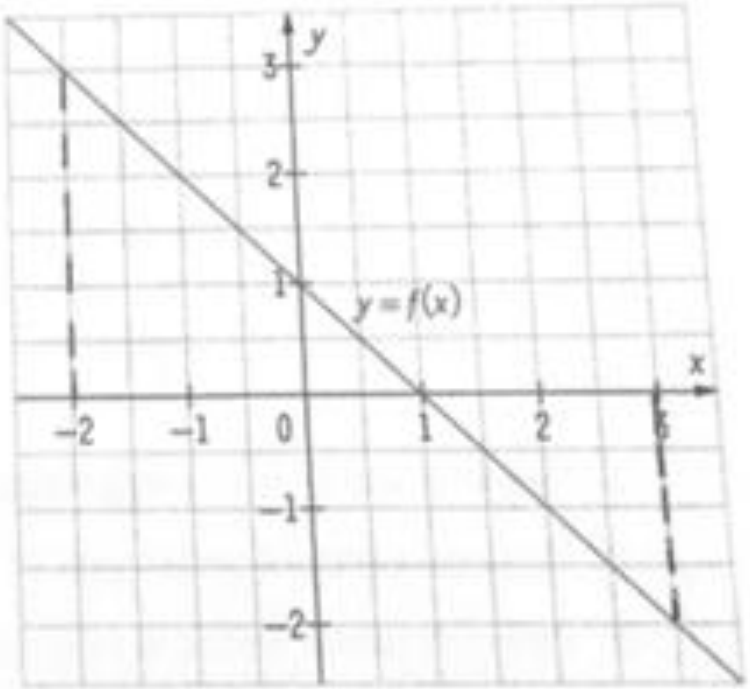
D $\frac{5}{2}$

E $\frac{13}{2}$

$\frac{(3 \cdot 3)}{2} = 4,5$

$\frac{(2 \cdot 2)}{2} = -2$

$4,5 - 2 = 2,5$

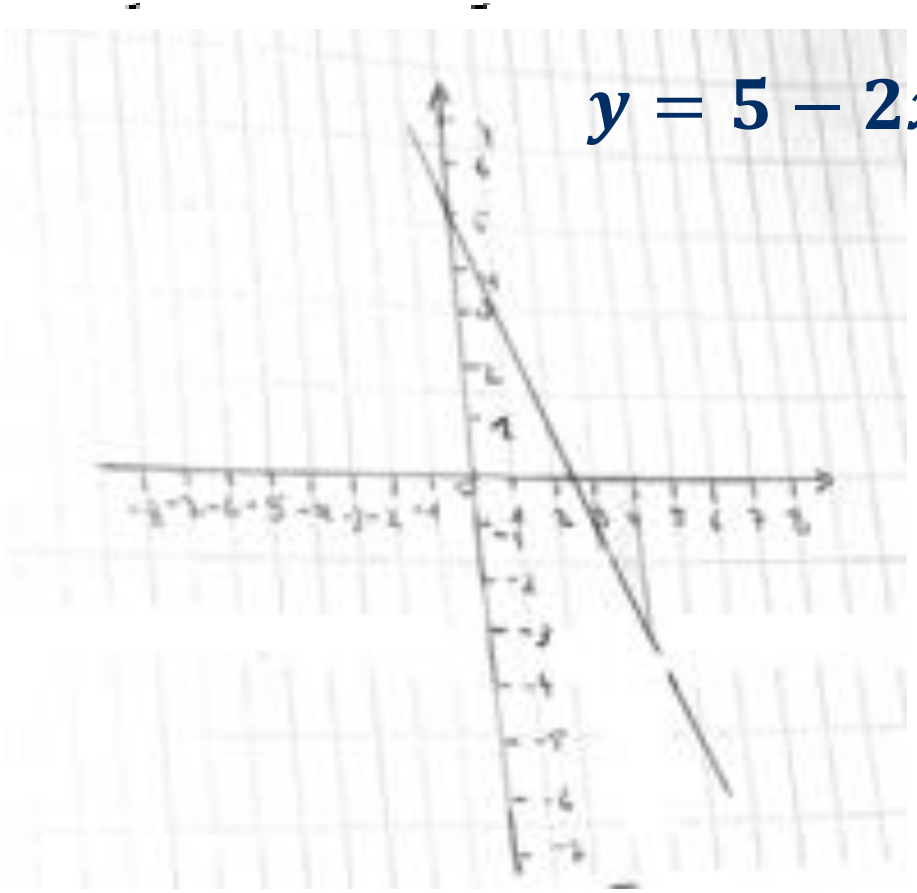


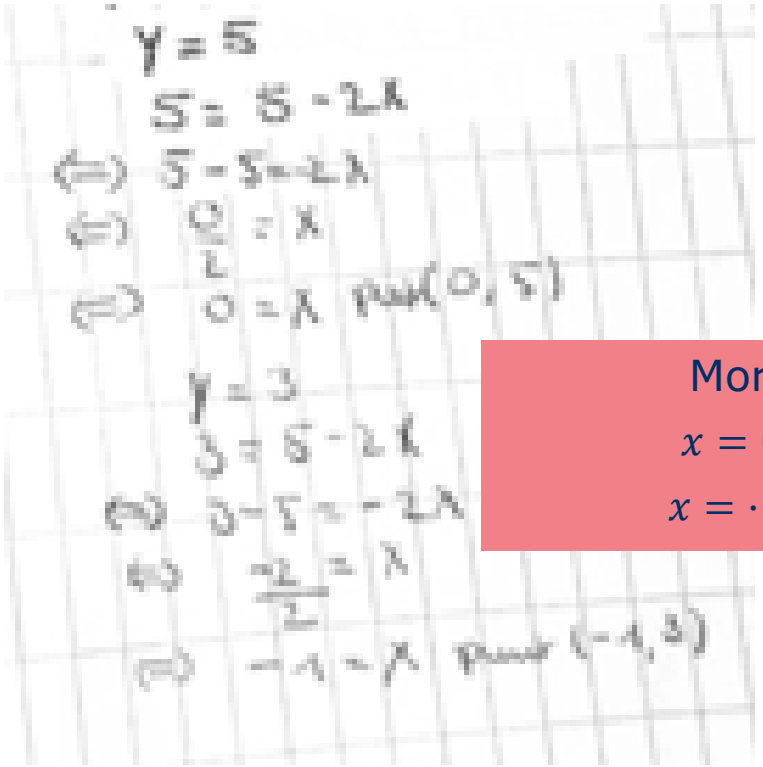
Use of pictures of exercises of learners

Intro – Exit examples

Review of exercise:
method not efficient

Describe how to find two points af a straight line more efficient





More efficent:

$x = 0 \Rightarrow y = 5$

$x = \cdots \Rightarrow y = \cdots$

Repeat at start - during - end of lesson

- **New subject matter**
- **Subject matter previous lesson**
- **Subject matter previous year**

EXIT-TICKET **NAAM:**

Oefening 1

Gegeven de punten $A(-2,3)$ en $B(4,-5)$, wat zijn de coördinaten van het midden van het lijnstuk $[AB]$?

Oefening 2

Gegeven de punten $A(3,1)$ en $B(1,2)$, wat zijn de coördinaten van $(2 \cdot \vec{B} - \vec{A})$?

Oefening 3

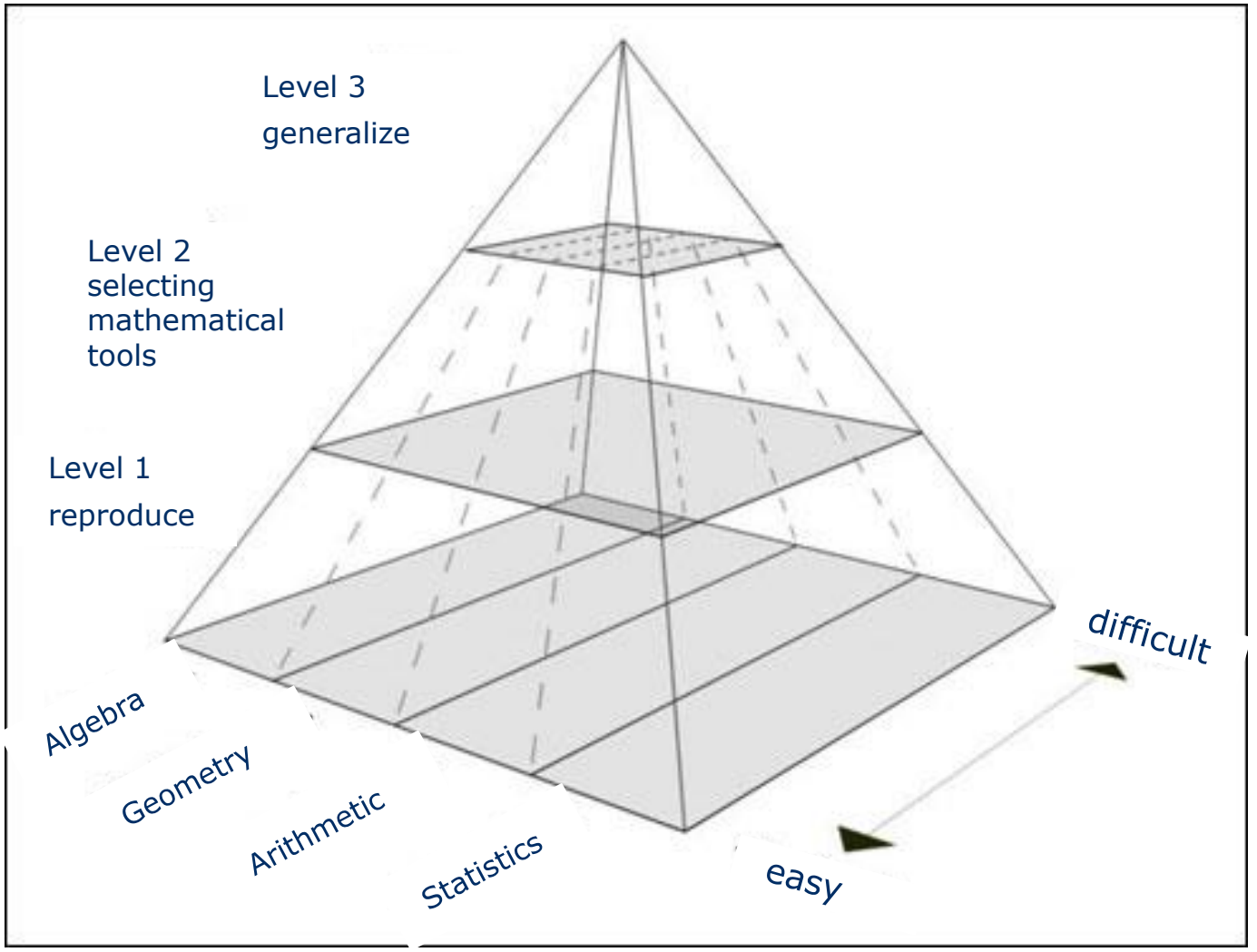
Gegeven de punten $A(2,3)$ en $B(3,-3)$, wat zijn de coördinaten van de vector $\overrightarrow{AB}$?

Variation in questions : Test pyramid

Insight skills reflection

Insight reflection

Knowledge



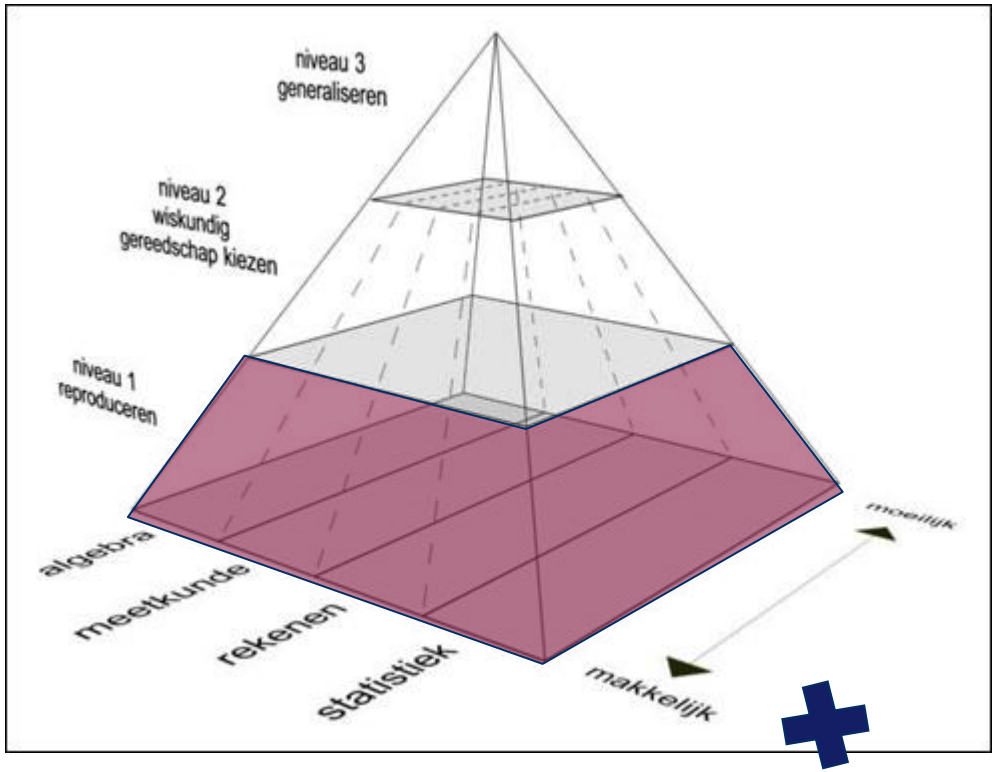
Handboek wiskunde did. p317

Variation in objectives => variation in questions

Solve the following equation:

$$-3x^2 + 2x + 1 = 0$$

$$\Leftrightarrow x = -\frac{1}{3} \text{ V } x = 1$$

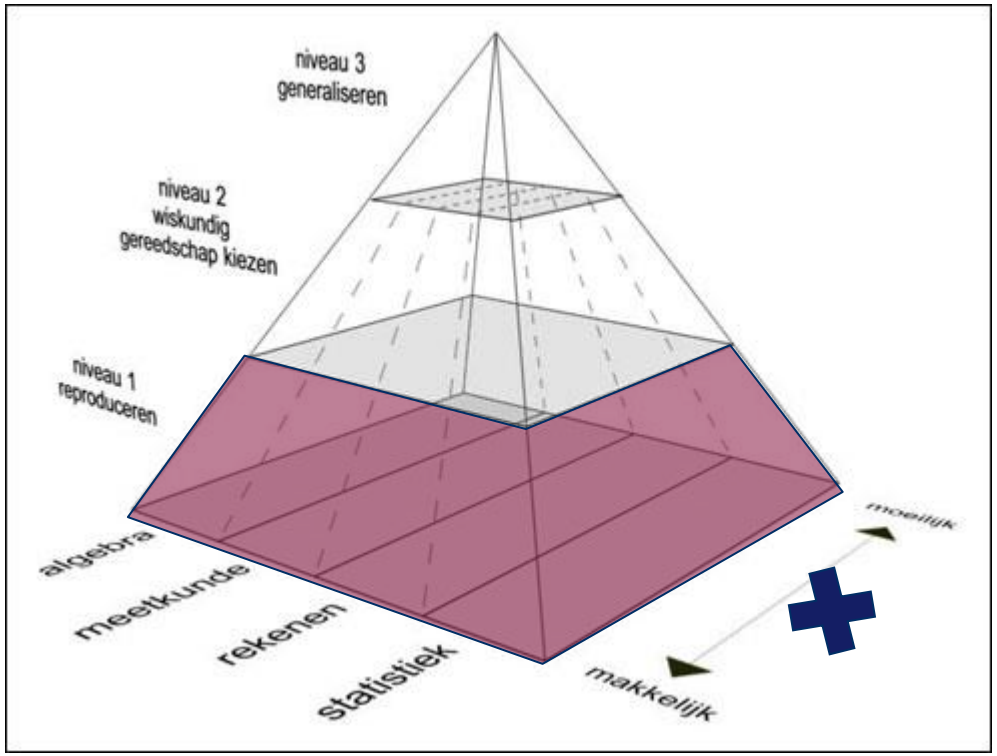


more simple: $3x^2 - 2x - 1 = 0$

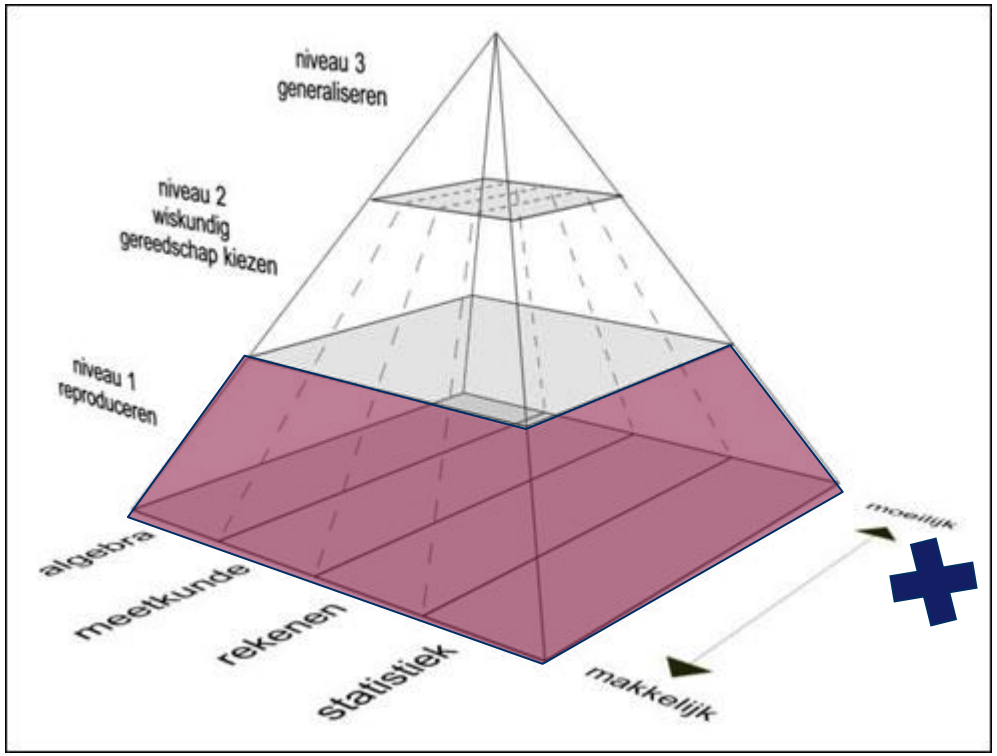
Variation in objectives => variation in questions

Solve the following equation :

$$4x - 5 = 6x^2 - 7$$
$$\Leftrightarrow -3x^2 + 2x + 1 = 0$$
$$\Leftrightarrow x = -\frac{1}{3} \text{ V } x = 1$$



Variation in objectives => variation in questions



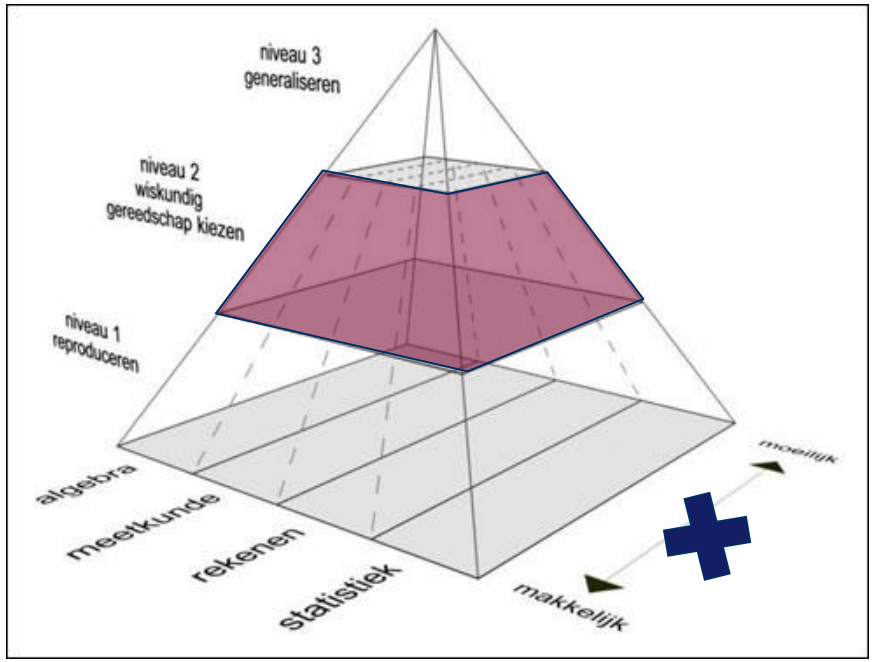
Solve the following equation :

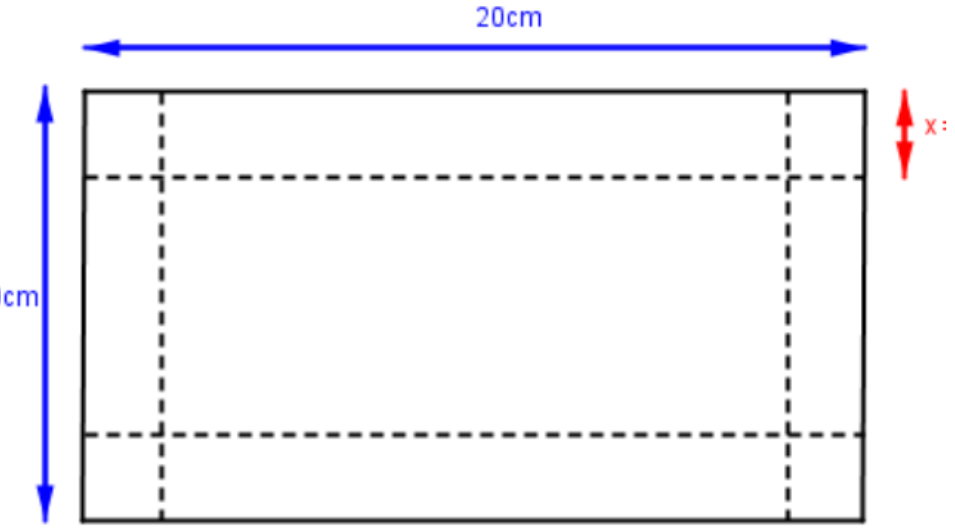
$$3x - 6 = 5x^2 - 24$$

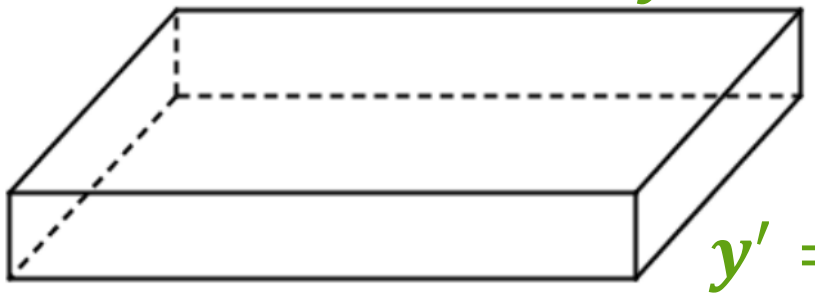
$$\Leftrightarrow x = -1{,}62 \ldots \text{ v } x = 2{,}22 \ldots$$

Variation in objectives => variation in questions

Kaan wants to make a rectangular box with height 10cm and circumference 20cm from a piece of cardboard. What dimensions should the cardboard have in order for the volume to be as large as possible?







$$y = x(20 - 2x)(10 - 2x)$$

$$y = 200x - 60x^2 + 4x^3$$

$$y' = 12x^2 - 120x + 200$$

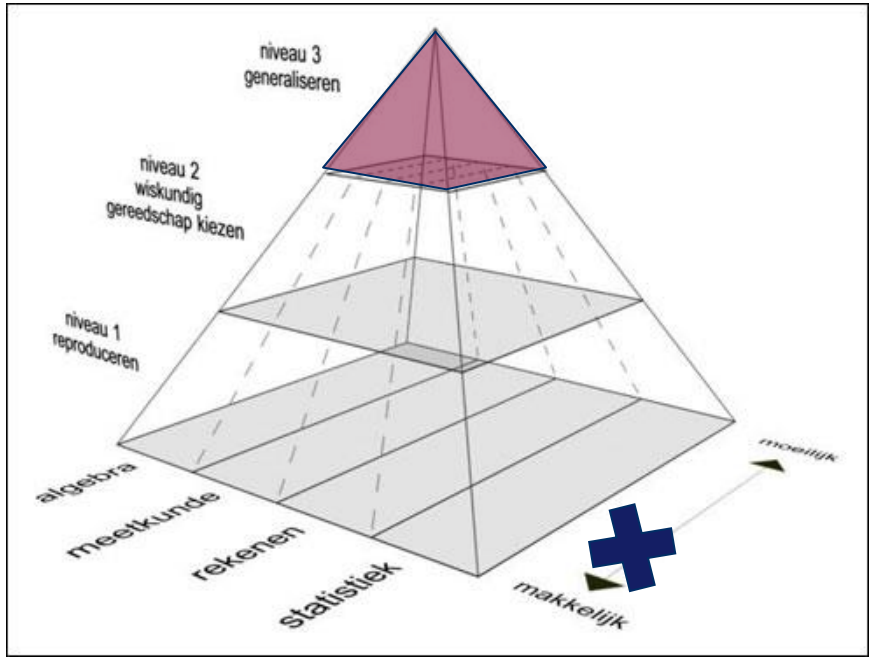
Variation in objectives => variation in questions

Calculate for what value(s) of p

the functions

$f(x) = 4{,}5x^2 + px$ en $g(x) = -2$

have an intersection point.

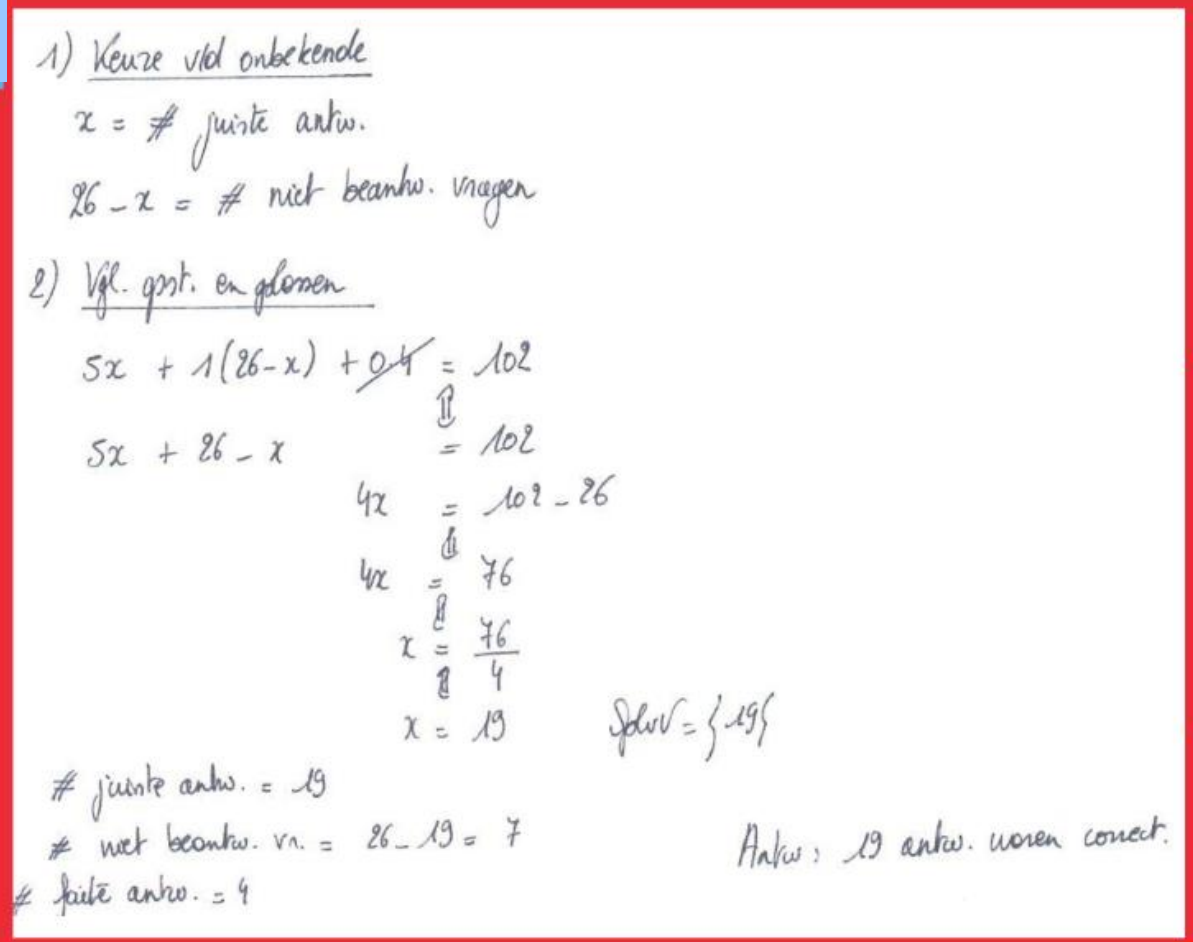


Learning to solve difficult questions: How?

- **Learning to start with solving**
- **Question (challenging) part out of the solution**
- **Test insight via short questions that can be used as**
  - Intro ticket
  - Question after a lesson sequence
  - Exit ticket
- **Closed multiple choice questions**
- **From open or long procedure to closed question (recap day 2 part II)**

Avoid stacked questions

The Junior Mathematics Olympiad consists of 30 multiple-choice questions. For each correct answer, you earn 5 points. Of course, you get no points for each incorrect answer, but for each unanswered question, you earn 1 point. Jurgen scored 102 points and gave 4 incorrect answers. How many answers were correct?



3 questions:

- Modeling
- Solving the equation
- Giving final answer

The learner cannot continue if the modeling step is incorrect.

Not always wrong, but often is.

Question (challenging) part out of the solution

Alternatief 1

Vraag 1

Solve _ in $\mathbb{Q}$:

$$5x + \frac{5}{2} = \frac{10}{3}x$$

Vraag 2

De Junior Wiskunde Olympiade telt 30 meerkeuzevragen. Voor elk juist antwoord krijg je 5 punten. Voor elk verkeerd antwoord krijg je uiteraard geen punten, maar voor elke niet beantwoorde vraag krijg je 1 punt. Zo heeft Jurgen 102 punten behaald en 4 foute antwoorden gegeven. Hoeveel antwoorden waren correct?

Set up an equation that can solve this problem.

You should NOT solve the equation.

part of solution

Alternatief 2

De Junior Wiskunde Olympiade telt 30 meerkeuzevragen. Voor elk juist antwoord krijg je 5 punten. Voor elk verkeerd antwoord krijg je uiteraard geen punten, maar voor elke niet beantwoorde vraag krijg je 1 punt. Zo heeft Jurgen 102 punten behaald en 4 foute antwoorden gegeven. Hoeveel antwoorden waren correct?

If you can't find the equation, ask your teacher for it.

Alternatief 3

De Junior Wiskunde Olympiade telt 30 meerkeuzevragen. Voor elk juist antwoord krijg je 5 punten. Voor elk verkeerd antwoord krijg je uiteraard geen punten, maar voor elke niet beantwoorde vraag krijg je 1 punt. Zo heeft Jurgen 102 punten behaald en 4 foute antwoorden gegeven. Hoeveel antwoorden waren correct?

Show algebraically that 19 is a correct answer. Formulate the answer forming a good sentence

Short questions to test insight (intro - in les – exit)

Objective: The learner solves a system of equations

(7 punten) Set up the extended matrix of the following system of equations.

$$\begin{cases} 4x - 3y - 5z = 25 \\ x + 5y - 7z = 12 \\ 6x + y - 13z = 43 \end{cases}$$

Setting op the matrix is the first step when using the method of Gauss-Jordan

Short questions to test insight (intro - in les – exit)

1) Solve the matrix with G-J into a 'resolved matrix':

$$\left[\begin{array}{ccc|c} 5 & 10 & 15 & 50 \\ 15 & 20 & 0 & 50 \\ 10 & 10 & 10 & 50 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

2) Note the solution set of the system of equations with the following' resolved matrix':

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & -3 & 5 \\ 0 & 0 & 1 & 0 & -11 & 2 \\ 0 & 0 & 0 & 1 & 4 & -7 \end{array}\right]$$

$$V = \{(5 - 2r + 3s, 2 + 11s, -7 - 4s) | r, s \in R\}$$

8a $\cos(\pi-\alpha)=-\cos\alpha$

8b $\tan(\pi+\alpha)=\tan\alpha$

8c $\sin(-\alpha)=-\sin\alpha$

8d $\cos(\alpha-\frac{\pi}{2})=\sin\alpha$

8e $\sin(3\pi-\alpha)=\sin\alpha$

8f $\cot(\frac{\pi}{2}-\alpha)=\tan\alpha$

Short questions to test insigh: Knowledge =>Insight

Possible extra question to differentiate:

8g Simplify the expression:

$$\frac{\cos(\pi-\alpha).\tan(\pi+\alpha).\sin(-\alpha)}{\cos(\alpha-\frac{\pi}{2}).\sin(3\pi-\alpha).\cot(\frac{\pi}{2}-\alpha)}=$$

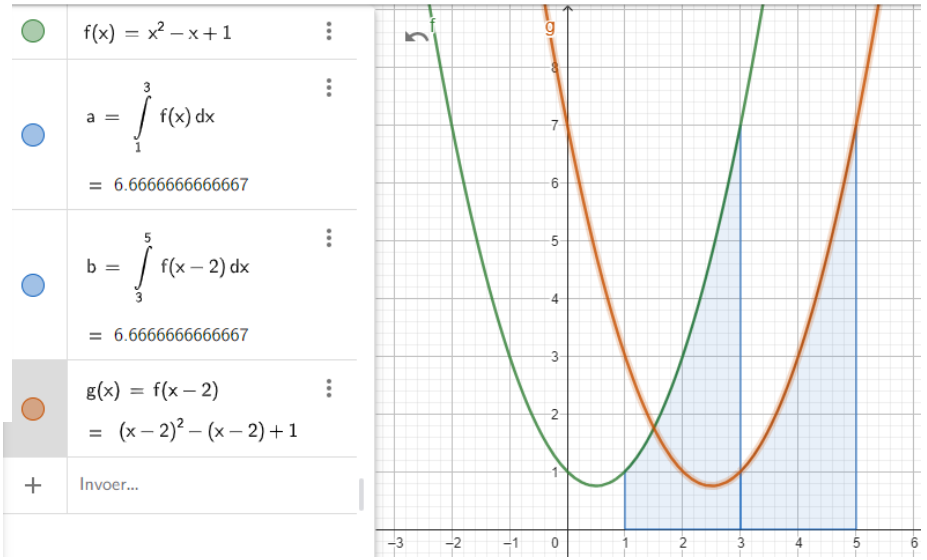
$$\frac{-\cos(\alpha).\tan(\alpha).(-\sin\alpha)}{\sin(\alpha).\sin(\alpha).\tan(\alpha)}$$

$$=\frac{\cos(\alpha)}{\sin(\alpha)}(eventueel=\cot\alpha)$$

Closed multiple choice questions

Gegeven dat $\int_1^3 f(x)dx = 2{,}4$ Wat is dan $\int_3^5 f(x-2)dx$?

a) 0,4
b) 2,4
c) 4,4



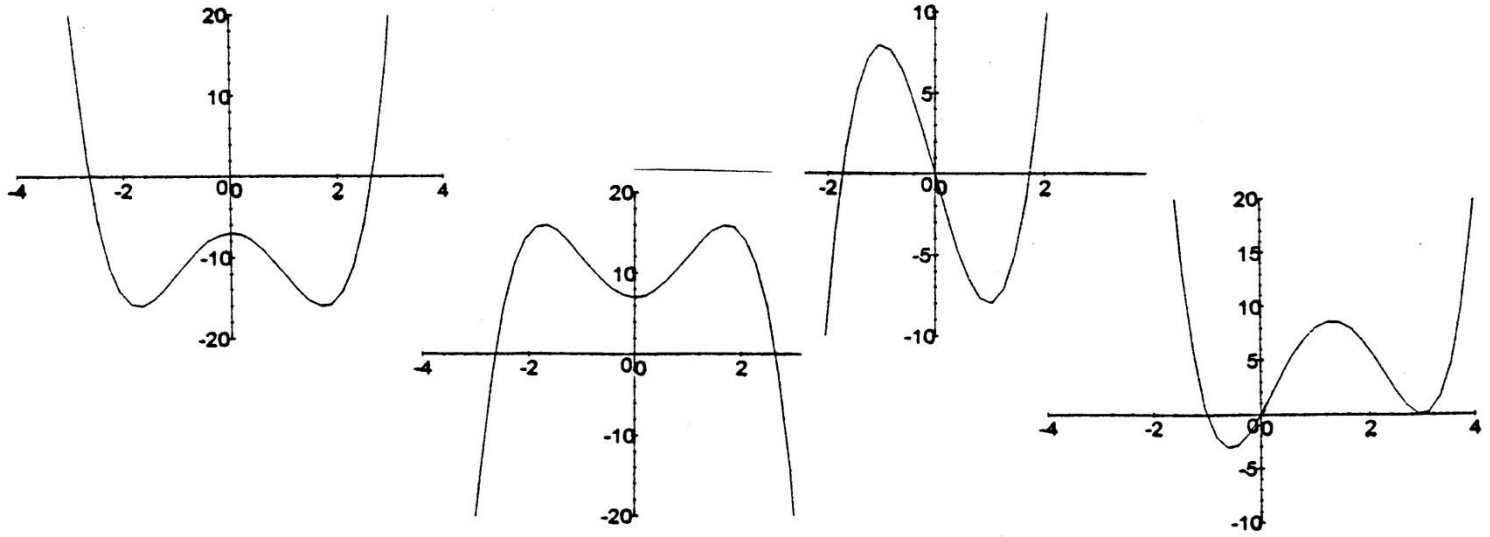
A challenge to develop quality MC questions that test insight

Closed multiple choice questions

Given the sign chart:

x		$-\sqrt{3}$		-1		0		1		$\sqrt{3}$	
$f'(x)$	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$
$f''(x)$	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$	$+$	$+$
$f(x)$	...	...	...	...	...	...	...	...	...	...	...

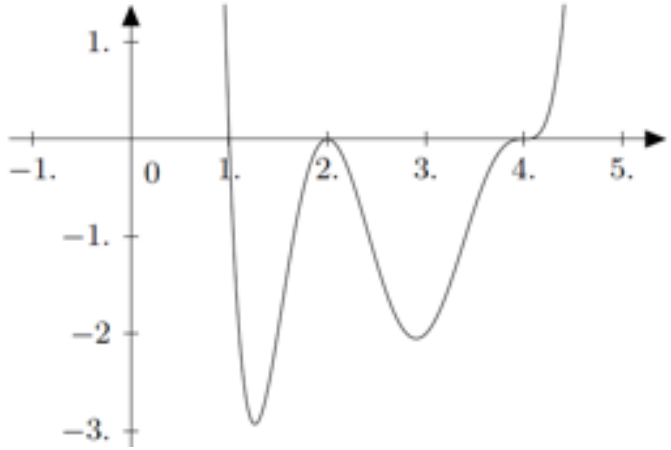
Which graph matches?



Closed multiple choice questions (True – false)

Deeper thinking questions

Consider the graph. Which operations are true or false



	True	False
a) 2 is a root with even multiplicity.	✖	
b) 4 is a simple root.		✖
c) 1 is a simple root.	✖	
d) The degree of this polynomial function is at most 3.		✖
e) The polynomial function can have degree 10.		✖
f) The polynomial function can have degree 5.	✖	

Explain meaning

‘The integral is an infinite sum.’

To this statement, we find the following two formulas in the course:

$$\text{if } n \to \infty \text{ then } f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x \to \int_a^b f(x)dx$$

Explain the meaning of 1 term of the sum.

Instead of: give the definition of …

Explain meaning

Each function $f: \mathbb{R} \to \mathbb{R} : x \mapsto ax^2 + bx + c$ where $a \in \mathbb{R}_0$ and b, c $\in \mathbb{R}$, is a *second degree function.*

Why does this definition exclude that $a = 0$?

Explain meaning

If Elif obtains the solution set $V = \left\{\frac{2\pi}{3} + k \cdot \frac{\pi}{2}\right\}$ for a trigonometric equation and Tigran obtains $V = \left\{\frac{5\pi}{3} - k \cdot \frac{\pi}{2}\right\}$, do they then obtain the same solutions? Explain in your own words.

Assessing Solutions

Question: For which value of x is the following statement true:

$$5x - 10 = -3x + 30$$

Below are the answers of 3 learners. Are the answers right or wrong? Why?

$5x - 10 = -3x + 30$	$5x - 10 = -3x + 30$	$5x - 10 = -3x + 30$
$2x - 10 = 30$	$8x + 10 = 30$	$8x - 10 = 30$
$2x = 20$	$8x = 20$	$8x = 40$
$x = 10$	$x = 2{,}5$	$x = 5$

Handboek wiskunde didactiek p313

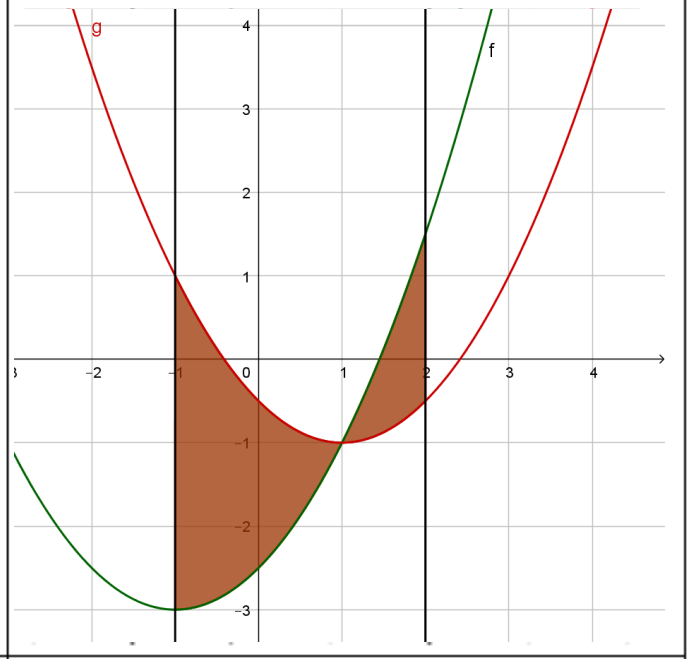
Evaluate solution - justify?

Question about finding the shaded area

Evaluate the solution. Give an explanation whereby you use the meaning of the integrals and the given graphs.

Instead of:

Solve

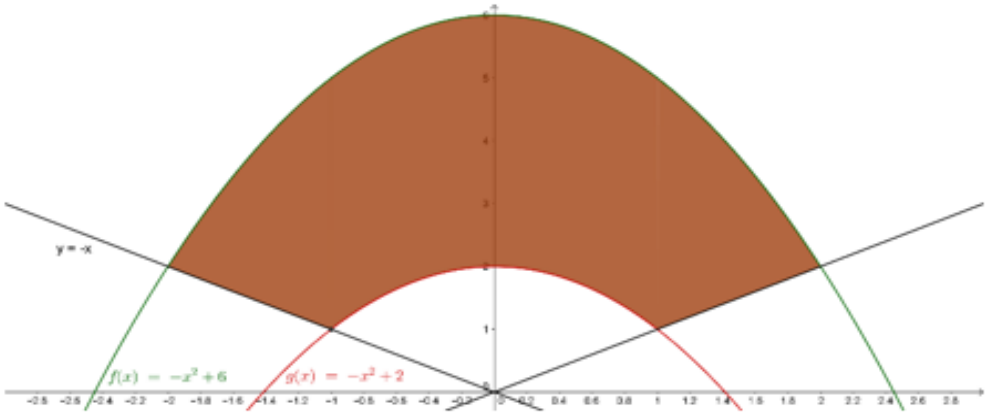
Oplossingsmethode en verklaring	Ondersteuning verklaring
$\int_{-1}^{1} [g(x) - f(x)]\,dx + \int_{1}^{2} [f(x) - g(x)]\,dx$ Juist/fout?: Verklaring:	

Efficient methods

Repeat

Gegeven zijn de volgende functies: $y = x$, $y = -x$, $f(x) = -x^2 + 6$ en $g(x) = -x^2 + 2$.

a. Bereken de coördinaten van de punten die je nodig hebt om de oppervlakte te berekenen van het gearceerde gebied. Duid de punten aan op de grafiek. (1p)



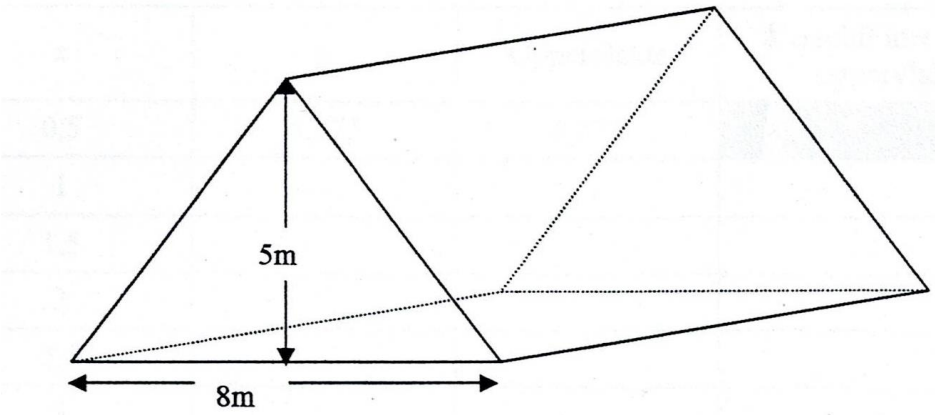
b. Leg eerst uit waarom de onderstaande integralen de oppervlakte van het gearceerde gebied berekenen. Beargumenteer vervolgens of dit een efficiënte methode is of niet. (2p)

$$2\left[\int_0^1 \left(-x^2 + 6 - (-x^2 + 2)\right)dx + \int_1^2 \left(-x^2 + 6 - (x)\right)dx\right]$$

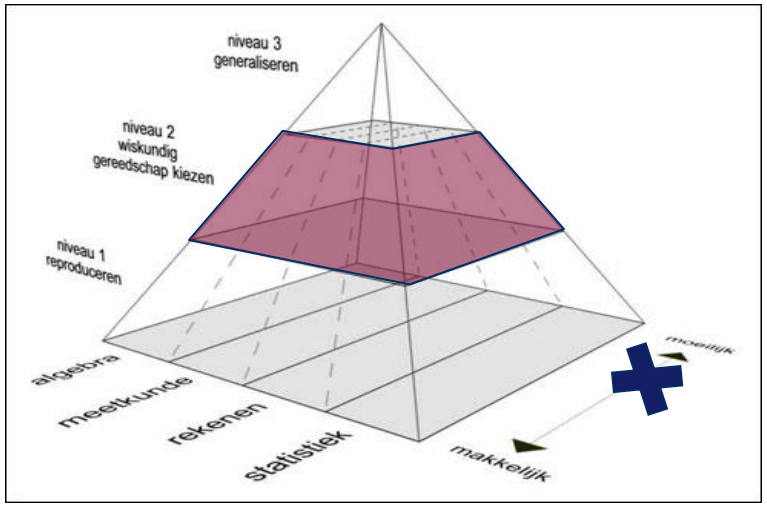
Make a step-by-step plan

Make a step-by-step plan toe solve the following exercise:

Kaan builds wooden bungalows of the shape below with height 5m, width 8m and length 10m. In the side wall there will be a rectangular glass door as large as possible. What are the dimensions of this glass door?



$$y = \frac{-5}{2}x^2 + 10x$$



Plan B: (hidden) stacked question $x^3 - 8x + 8 \geq 0$

For question 1 and 2 is given that $f(x) = x^3 - 8x + 8$

1. **Calculate algebraically the zero points of f**
2. **Make a sign table for f.**
3. **Explain why the sign table in 2 can be used to solve $x^3 - 8x + 8 \geq 0$.**

4. **Solve the inequality $x^3 - 8x + 8 \geq 0$**
5. **Given the functions $f(x) = x^3 - 4x$ and g$(x) = 4x - 8$, investigat for which values of x the graph of f is below the one of g**

From long procedure to closed Q $x^3 - 8x + 8 \geq 0$

For question 1 and 2 is given that $f(x) = x^3 - 8x + 8$

1. **Calculate algebraically the zero points of f**
2. **Make a sign table for f.** If you did not find the zero points in 1 you may use the calculator OR…
3. **Explain why the sign table in 2 can be used to solve $x^3 - 8x + 8 \geq 0$.** If you did not find the drawing chart, ask your teacher. Afterwards you may not work on 1 and 2
4. **Solve the inequality $x^3 - 8x + 8 \geq 0$**
5. **Given the functions $f(x) = x^3 - 4x$ and g$(x) = 4x - 8$, investigat for which values of x the graph of f is below the one of g**

Questions?

Task: (hidden) stacked questions

- Why is this a stacked question?
- Give alternatives for this question

Objective: The learner mathematizes horizontally

(5 punten) Twee vrienden, Jo & Mohamed, hebben knikkers.
Als Mohamed 1 knikker aan Jo geeft, heeft hij er dubbel zoveel als Jo.
Als Jo er 3 aan Mohamed geeft, heeft Mohamed er 4 maal zoveel als Jo.
Hoeveel knikkers hebben Jo & Mohamed?
Berekening met het grafisch rekentoestel. Formuleer een antwoord.

Task: (hidden) Stacked questions

Objective: The learner solves a system of equations.

Give the solution set for the system of equations

$$\begin{cases} 3x + 2y = 12 \\ 2x - y - 4 = 0 \end{cases}$$

- Why are this stacked questions?
- Give alternatives for these questions.

Objective: the learner uses graphs to solve a system of equations

Plot the graphs for the following set of equations, find the solution using these grapsh.

$$\begin{cases} 3x + 2y = 12 \\ 2x - y - 4 = 0 \end{cases}$$

Task : Use your exam or test questions

- **Check:**
  - Are there (hidden) stacking questions?
  - Do the questions test both knowledge and understanding?
  - Place the questions in the test pyramid
- **Ask short (insight) questions to the questions asked (recap: from open to closed questions)**