ProTEEM

TEAM UP FOR PROTEEM

Co-funded by the European Union

Module 2: Didactics in Mathematics – Day 2 Part II

August, 19-23, Antwerp

Mrs. Gilberte Verbeeck

Universiteit Antwerpen
ASOE | Antwerp School of Education

Exercise 1

- Duo assignment (20')
- One person solves the given problem: see info front page
- The other person observes and writes down which mathematical reflexes and methods are being used to solve the problem: see info back page

Exercise 1

Solve the following problem (20'):

The floor of a rectangular room is covered with (entire) square tiles. The room is m tiles wide and n tiles long. Half of the tiles are on the edge. For how many room sizes is this possible?

(A) none (B) 1 (C) 2 (D) 3 (E) more than 3

(Flemish Mathematics Olympiad volume 1996, 2e round, question 18)

Exercise 1

- **Solution?**

- **How did you arrive at this solution?**
- **How did you proceed?**

Problem-solving skills

- **In exercise 1 you used your problem-solving skills**

- **Problem = ?**
  - Task/question/exercise for which you do not immediately know how to find a solution

- **What is essential when solving problems?**
  - Based on your experiences?
  - According to Schoenfeld? (Referentie: *A. H. Schoenfeld, Mathematical problem solving, Academic press, 1985)*

Schoenfeld on problem solving

- **Ability to solve mathematical problems is based on...**

  Knowledge base, substantive background (resources)

  i.e. knowledge of definitions, properties, techniques and frequently used methods, but also insight and intuition into the learning material involved

- **BUT KNOWLEDGE ALONE IS NOT ENOUGH!**

Schoenfeld on problem solving

- **Ability to solve mathematical problems is based on...**

  Knowledge base, substantive background (resources)

  i.e. knowledge of definitions, properties, techniques and frequently used methods, but also insight and intuition into the learning material involved

- **BUT KNOWLEDGE ALONE IS NOT ENOUGH!**

- **ALSO NEEDED:**

  Problem solving heuristics

Schoenfeld on problem solving

Problem solving heuristics

- are **advice** that increases the chance of finding the solution, but without guarantee of success.
- are **general**: they do not relate to one specific piece of learning material
- e.g.
  - make a figure
  - enter good notations
  - structure the elements of the assignment
  - work from back to front
  - first consider a special case
  - view extreme cases
  - translate the problem into a more suitable representation
  - drop a condition
  - …

Schoenfeld on problem solving

- **Ability to solve mathematical problems is based on...**
  - **Knowledge base,** substantive background (resources)
  - **Problem solving heuristics**

- **BUT THIS ALONE IS NOT ENOUGH!**

Schoenfeld on problem solving

- **Ability to solve mathematical problems is based on...**
  - **Knowledge base,** substantive background (resources)
  - **Problem solving heuristics**

- **BUT THIS ALONE IS NOT ENOUGH!**

- **ALSO NEEDED:**
  - **Monitoring and control** (Metacognition)

Schoenfeld on problem solving

- **Monitoring and control (Metacognition)**

  Agile management of the solution process

  - Planning, interim evaluation, making decisions, ...
  - In this way, string together heuristics and substantive background

- **Why?**
  - Interpret heuristics (differently) for each problem
  - Select custom heuristics

Schoenfeld on problem solving

- **Ability to solve mathematical problems is based on...**
  - **Knowledge base,** substantive background (resources)
  - **Problem solving heuristics**
  - **Monitoring and control (Metacognition)**

- **BUT THIS IS STILL NOT ENOUGH!**

Schoenfeld on problem solving

- **Ability to solve mathematical problems is based on...**
  - **Knowledge base,** substantive background (resources)
  - **Problem solving heuristics**
  - **Monitoring and control (Metacognition)**

- **BUT THIS IS STILL NOT ENOUGH!**

- **ALSO PLAY A ROLE:**

  Beliefs and **feelings**

Schoenfeld on problem solving

- **Opinions, beliefs**
  - about the subject of mathematics
    - e.g. congruence cases of triangles in the proofs from the lesson BUT ALSO as an aid in searching for a problem
    - Using mathematical knowledge = **believing in its usefulness**
  - about what is expected of students
    - e.g. giving up searching too quickly because of the belief that '**finding something yourself = genius**'
  - about...
- **Feelings**
  - e.g. discouragement if it doesn't work immediately
  - e.g. confidence in one's own abilities

Schoenfeld on problem solving

- **Take these views and feelings into account!**

- **Views and feelings can change due to experiences, e.g**
  - attention to the **usability** of mathematics
  - to gain **success experiences** in solving problems
  - being able to solve problems yourself = growth in **self-confidence**
  - provide **feasible, challenging assignments (differentiate!)**
  - learning that mathematics is not "finished" - every search process can lead to new, interesting things

How?

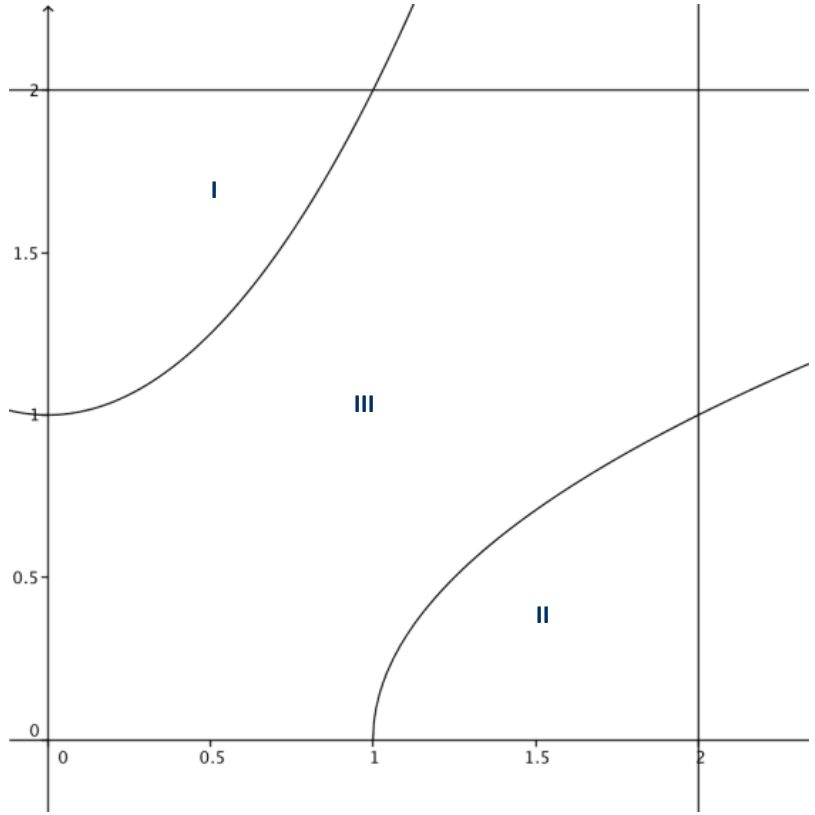
Example: exercise 2

- In the figure you see the graphs of $f(x) = x^2 + 1$ and g(x) = $\sqrt{x-1}$ and the lines $x = 2$ and $y = 2$.

  Three areas can be distinguished: I, II and III.

- Determine the area of each of the three areas I, II and III in (at least) two different ways.

Be creative!



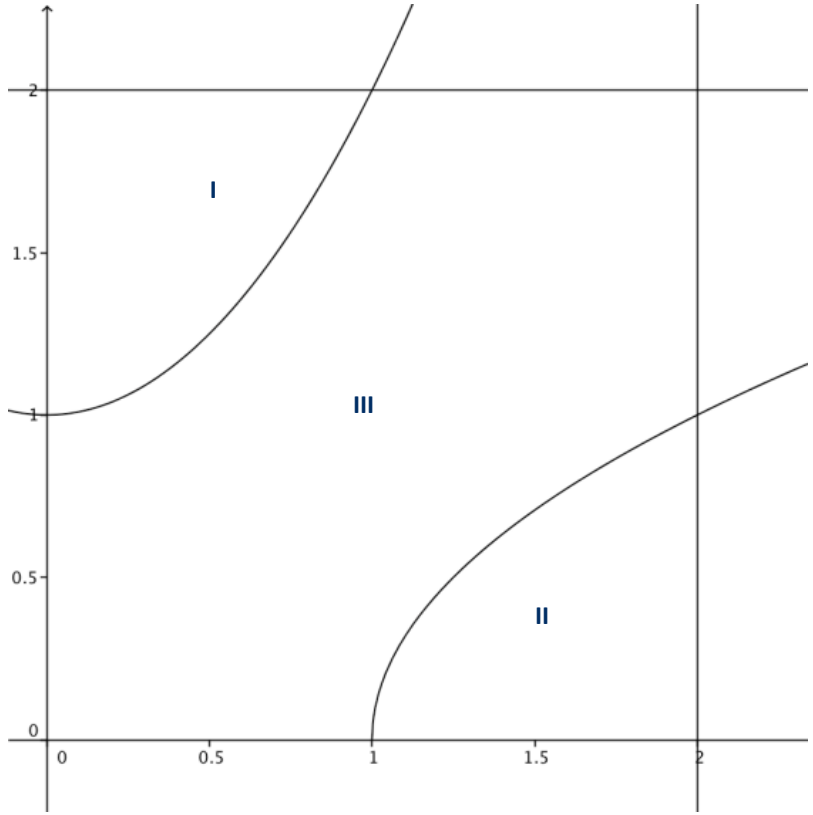
Solutions?

Exercise 2: revised

In the figure you see the graphs of $f(x) = x^2 + 1$ and g(x) = $\sqrt{x-1}$ and the lines $x = 2$ and $y = 2$. Whitin these formulas, three aras can be distignguished within the square formed by the positive x-axis and the y-axis and the lines $x = 2$ and $y = 2$: I, II and III.

a) Argue that the area size of area I is equal to the area size of area II.

b) Determine the area of each of the three areas I, II and III in (at least) 2 different ways. Be creative!

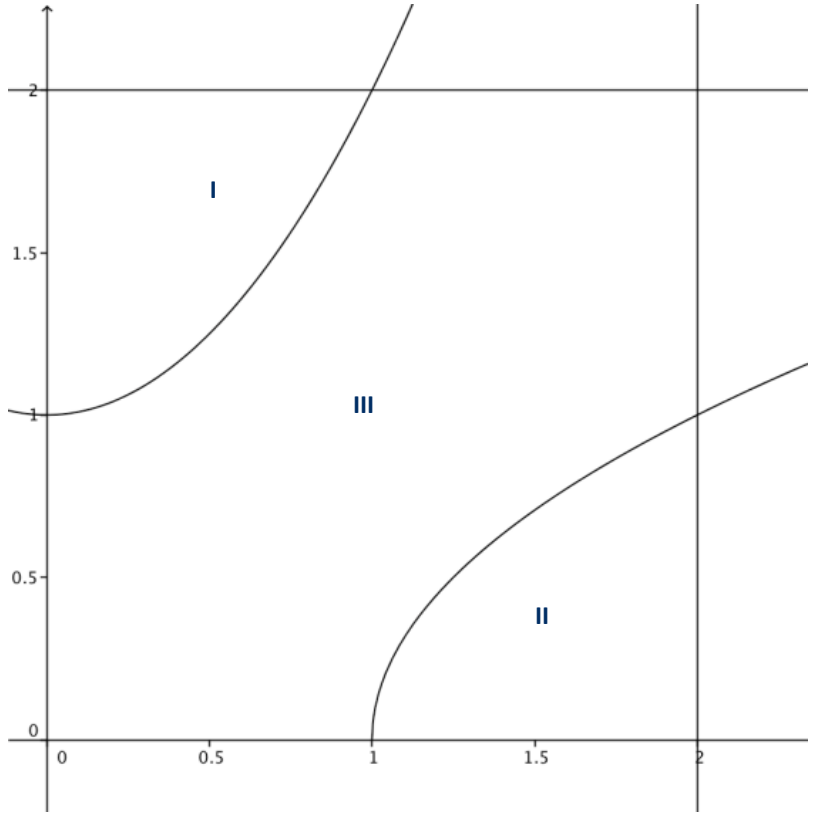


Exercise 2: revised

In the figure you see the graphs of $f(x) = x^2 + 1$ and g(x) = $\sqrt{x-1}$ and the lines $x = 2$ and $y = 2$. Whitin these formulas, three aras can be distignguished within the square formed by the positive x-axis and the y-axis and the lines $x = 2$ and $y = 2$: I, II and III.

a) Argue that the area size of area I is equal to the area size of area II.

b) Determine the area of each of the three areas I, II and III in (at least) 2 different ways. Be creative!



Tip: use your answer to question a) and properties of functions such as inverse function, symmetry, etc.

What do you notice?

Conclusion

- Provide feasible, challenging assignments by sufficiently differentiating!
  - Cf. problems with open vs. closed formulation

- Can you learn 'problem solving'?
  - To some extent
  - Provide a closed variant of assignment for students whose problem-solving skills have not yet been developed strongly enough

Assignment

- Have a look at the English mathematics textbooks
- Select an exercise.
- If it is an open variant of assignment, provide a closed variant of assignment for students whose problem-solving skills have not yet been developed strongly enough
- If it is a closed variant of assignment, provide an open variant of assignment for students whose problem-solving skills have already been developed strongly enough