

ProTEEM Day 2 Part II: Task – Problem Solving

Exercise 1

- **Duo assignment** (20')
- One person solves the given problem: see info below at the **front page**
- The other person observes and writes down which mathematical reflexes and methods are being used to solve the problem: see info at the **back page**

Solve the following problem:

The floor of a rectangular room is covered with (entire) square tiles. The room is m tiles wide and n tiles long. Half of the tiles are on the edge. For how many room sizes is this possible?

(A) none (B) 1 (C) 2 (D) 3 (E) more than 3

(Flemish Mathematics Olympiad volume 1996, 2e round, question 18)

Mathematical reflexes and skills (feel free to further supplement the list)

1. Working with concrete cases: reasoning based on an example
2. Predict. At the start of the problem you can think about which 'mathematical object' the result should be: a number with or without units, a ratio, a function, a yes/no answer, a proof... You can refer back to this when formulating the decision.
3. Check whether your findings are not too specific (e.g. only for the chosen example)
4. A property is not valid as soon as you can give one counterexample. A property is only valid if it applies in all situations.
5. When checking a property, you should do this on different types of examples.
6. Checking examples to demonstrate a statement is not enough. In a further phase you also need general proof.
7. Generalize by introducing variables.
8. Building a logical reasoning
9. Search for errors and analyze them
10. Check whether I am doing the right thing by looking back: does my approach lead to the answer to the question?
11. Be critical: Have I covered all options? Can I find an explanation for the results found? Are the formulas used correct?
12. Drawing up a table
13. Interpretation of an answer (can I have all possible values for a number, does this function exist everywhere...)
14. Using symmetry
15. Use of spatial insight
16. Consider what knowledge I can use: When browsing through your knowledge, you think about which parts of mathematics you can use to solve the problem (algebra, geometry, analysis, etc.) and more specifically which theorems, properties you can use (congruent triangles, Pythagoras, solving a system, making a graph, solving inequality...)
17. You can first tackle a simple version of the problem, then try to generalize it.
18. Reverse reasoning
19. When drawing up a proof, the following technique may help you further: 'start from the given, start from what needs to be proven, work in both directions at the same time, until both arguments 'meet' each other'.
20. A drawing (on paper or with a dynamic drawing package) is not a proof. If you draw a geometric situation accurately and as generally as possible, you can extract clues from it for a proof.
21. Using materials: sometimes you can try geometric problems with paper and scissors.
22. Making forecasts
23. Working psychologically (whether or not to have confidence in what you are doing)
24. Correct notations are important in reporting: $3+2=5+7=12$ is an example of what not to do.
25. Heuristics: drawing a picture, knowing and looking at a related problem, analyzing data/request, wording the problem differently, which data are redundant, solving part of the problem
26. Try and error
27. ...