ProTEEM

TEAM UP FOR PROTEEM

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Module 2: Didactics in Mathematics – Day 2 Part I

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Why should students learn math?

- **mathematics to use in everyday life**

  arithmetic from primary school, elementary geometric knowledge, some descriptive statistics, awareness of variability in sample results, ...

- **mathematics for use in other disciplines**

  algebraic arithmetic, eigenvalues and eigenvectors, first degree functions, exponential functions, derivation and integration, differential equations, testing hypotheses, ...

- **mathematics for its own sake**

  proofs, axiomatic and deductive work, mathematical structures, problem solving, beautiful geometric shapes, mathematics as a cultural object, ...

Which students is this math intended for?

- **mathematics to use in everyday life**

  Everyone, but especially young students and students who receive little mathematics

- **mathematics for use in other disciplines**

  future scientists (exact AND humane!), economists, statistics users, technicians, …

- **mathematics for its own sake**

  future mathematicians, anyone who wants to be inspired by the way mathematics works

How can we shape education so that students learn about the ubiquity, applicability and beauty of mathematics?

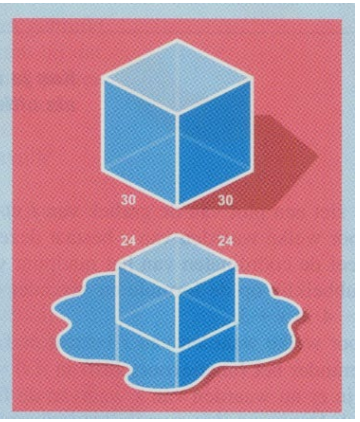
Mathematical applications

- **4 exercises:**

A

T_4 Een ijsblokje met ribben van 30 mm begint langzaam te smelten. Elke minuut worden de ribben 1,5 mm korter. Het volume van het ijsblokje wordt beschreven door de formule $V = (30 - 1{,}5t)^3$. Hierin is V het volume in kubieke millimeter en t de tijd in minuten.

a Bereken het volume van het ijsblokje op $t = 0$.
b Wat zijn zinvolle waarden voor t? En voor V?
c Plot en schets dat gedeelte van de grafiek waar beide variabelen betekenis hebben.
d Volg met de cursor de grafiek en onderzoek na hoeveel minuten het volume kleiner dan 10 000 mm^3 is. Geef je antwoord in 1 decimaal nauwkeurig.



B

3 De temperatuur in een koele berging wordt gegeven door de volgende functie:

$$T(t) = \frac{3t^2 - 6t + 3}{t^2 - 2t + 2}$$

T = de temperatuur in graad Celsius
t = tijd in uren
t = 0 komt overeen met 3 uur 's nachts

a Schets de grafiek van deze functie.
b Als de temperatuur lager wordt dan 1 °C is er gevaar voor schade aan het voedsel. Hoe lang bevond de temperatuur zich onder 1 °C? Van wanneer tot wanneer?
c Wanneer begon de temperatuur weer te stijgen?
d Naar welke temperatuur evolueert de koele berging?

C

Het grasveld van meneer Kok is 15 bij 20 meter. Meneer Kok besluit het grasveld te vergroten. Aan twee kanten komt er een even brede strook van x meter bij. Zie figuur 7.16.

a Toon aan dat de oppervlakte van het vergrote grasveld gegeven is door $opp = x^2 + 35x + 300$.
b Het nieuwe grasveld heeft een oppervlakte van 374 m^2.
Stel een vergelijking op en bereken hoeveel meter de strook breed is.

Fig. 7.16

D

Meneer Kok heeft een grasveld van 16 m op 40 m. Zijn grasmachine maait 40 cm breed. Hij begint aan de buitenkant te maaien en volgt de omtrek. Na hoeveel rondjes is hij halfweg?

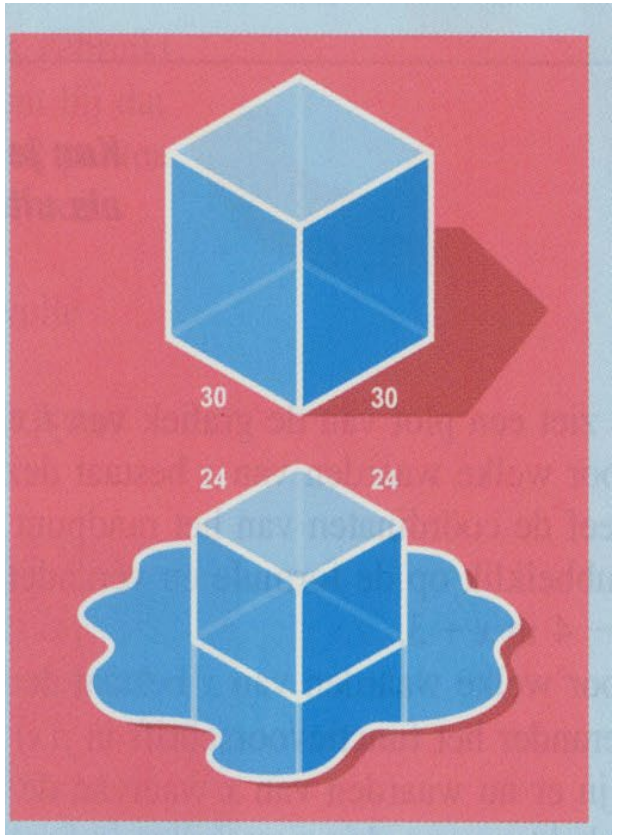
Exercises

- **3 (semi-)complementary groups:**
    1. Exercises A and D
    2. Exercises B and D
    3. Exercises C and D
- **Solve the two exercises (20')**
- **Each group will present one exercise (A, B or C) in plenary (10')**
- **Exercise D will also be presented and discussed in plenary (5')**

Exercise A

An ice cube with edges of 30 mm long starts to melt down slowly. Every minute, the edges get 1.5 mm shorter. The volume of the ice cube is described by the formula
$V = (30 - 1{,}5\ t)^3$, where V stands for the volume in mm^3 and t for the time in minutes.

a. Calculate the volume of the ice cube when t=0.
b. What are meaningful values for t? And for V?
c. Plot and sketch that part of the graph for which the variables are meaningful.
d. Trace the graph with the cursor and investigate after how many minutes the volume is less than 10 000 mm^3. Provide your answer with a precision of one decimal.



Exercise B

The temperature in a cool storage room is given by the following function:

$$T(t) = \frac{3t^2 - 6t + 3}{t^2 - 2t + 2}$$

T = temperature (°C); t = time (in hours); t = 0 corresponds to 3 a.m.

a. Sketch the graph of this function
b. If the temperature drops below 1 °C, there is a risk of damage to the food. How long was the temperature below 1 °C? From when to when?
c. When did the temperature start to rise again?
d. What temperature does the cool storage evolve to?

Exercise C

The lawn in Mr. Jones's garden measures 15 by 20 meters. Mr. Jones decides to extend the lawn. To two sides he adds a strip of equal width of x meters. See Figure 7.16.

a. Show that the area of the enlarged lawn is represented by $Area = x^2+35x+300$

b. The new lawn has an area of 374 m^2. Set up an equation and calculate the width of the strip.

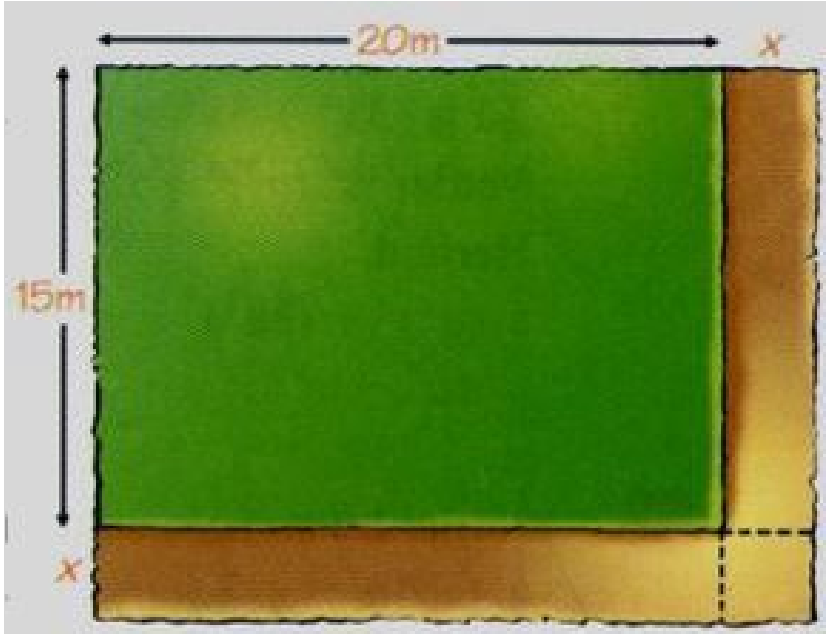


Fig. 7.16

Exercise D

Mr. Kok has a lawn of 16m by 40m. His lawn machine mows 40cm wide. He starts mowing on the outside and follows the perimeter. After how many laps is he halfway?

Compare exercises (5')

- Arrange exercises A, B, C and D of most to least appropriate to boost mathematical proficiency.
- Why do you think that assignment ... will boost the mathematical proficiency of students the most?
- Why do you think that assignment ... will boost the mathematical proficiency of students the least?
- What conclusion do you draw from this?

Mathematical applications

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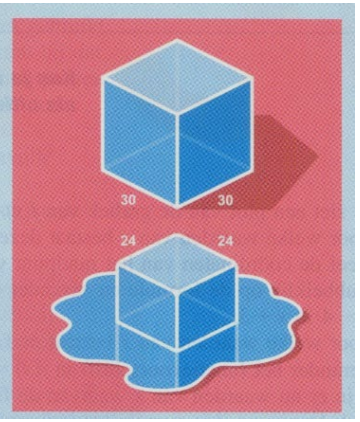

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Conclusion

- **'Packed' exercise (e.g. A, C):**
  - one started from a bare mathematical exercise
  - and comes up with an application-oriented cover for it
  - **does not contribute to a better understanding of reality**

- **Look for authentic problems (e.g. D) and integrate them regularly into your math lessons!**
  - are necessary to allow students to experience the **subtle relationship between reality and mathematics** and to learn to deal with it (e.g. critically considering the solution)
  - Allow students to see the ubiquity, applicability and beauty of mathematics and motivate them in the long run

Flashback

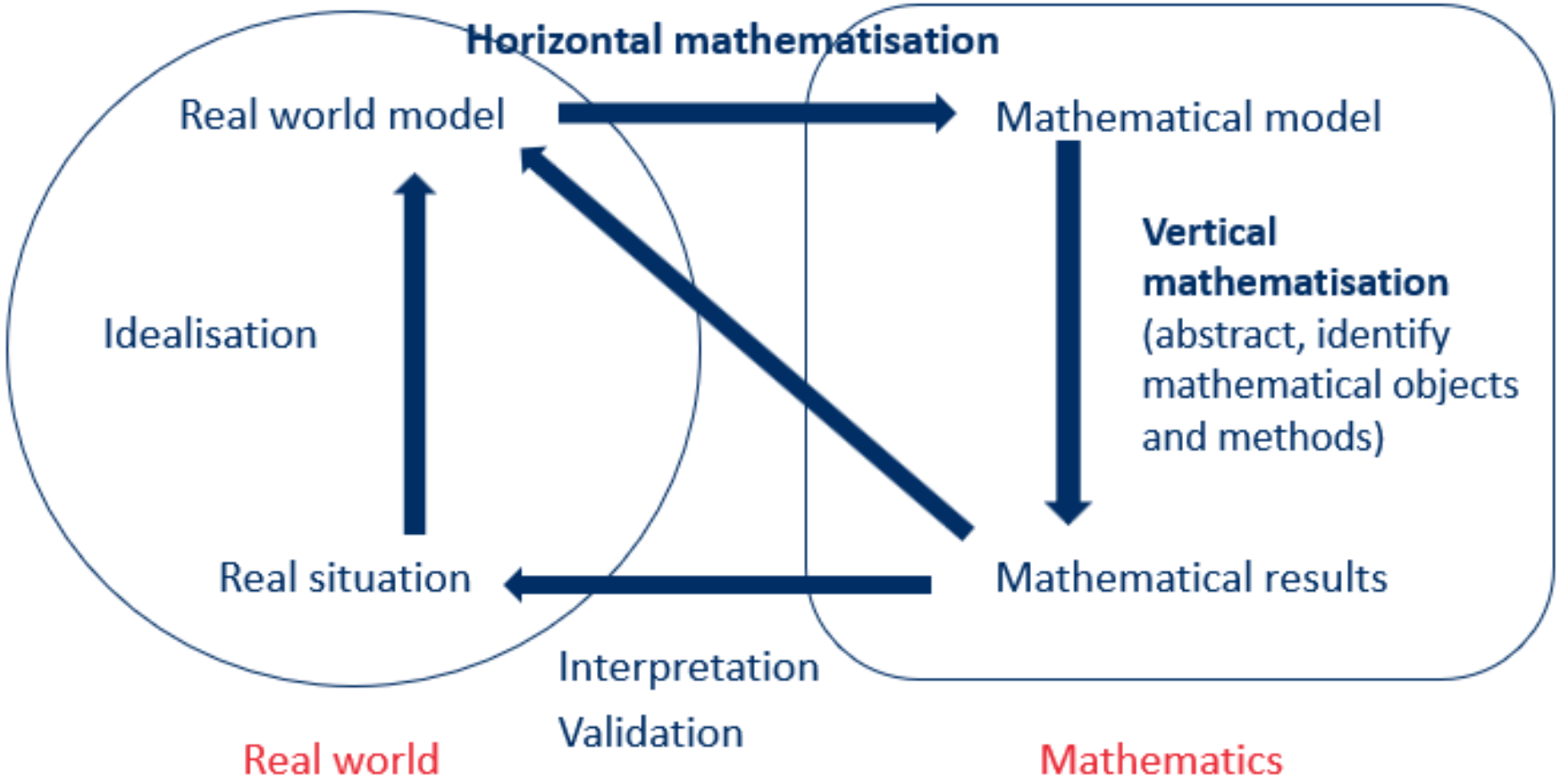
How can we shape education so that students learn about the ubiquity, applicability and beauty of mathematics?

Conclusion

- Find problems/phenomena that require a mathematical approach, where mathematics really helps (“Didactic phenomenology”)
- Connect mathematical trains of thought with phenomena in the physical, social, mental, ... world of students
- Make students realize how mathematics can help to organize and structure real-world problems
- As a teacher, take into account the skills, competencies and interests (world) of students
  - ➔ “**Reality Principle**” (one of the basic principles of Realistic Mathematics Education)
  - ➔ van de Heuvel-Panhuizen, M., & Drijvers, P. (2014). Realistic Mathematics Education, *Encyclopedia of Mathematics Education,* p.521-534.

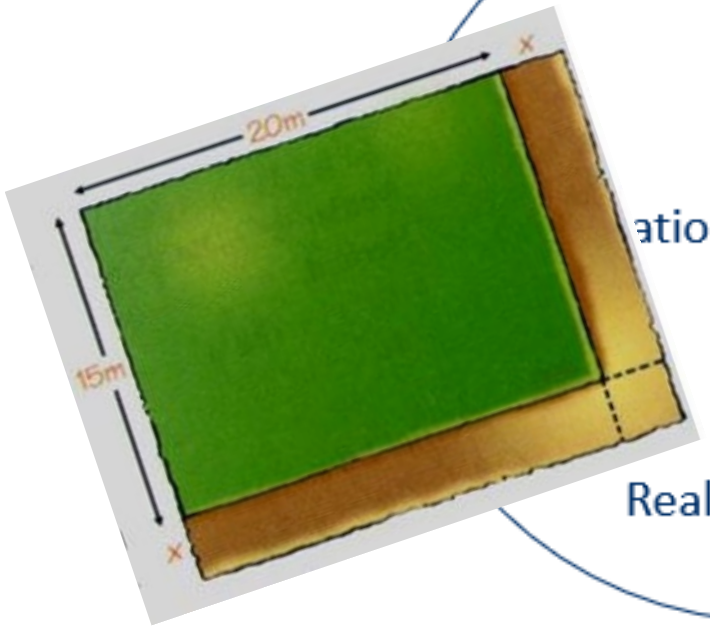
Analyze the thinking on assignment D (authentic)

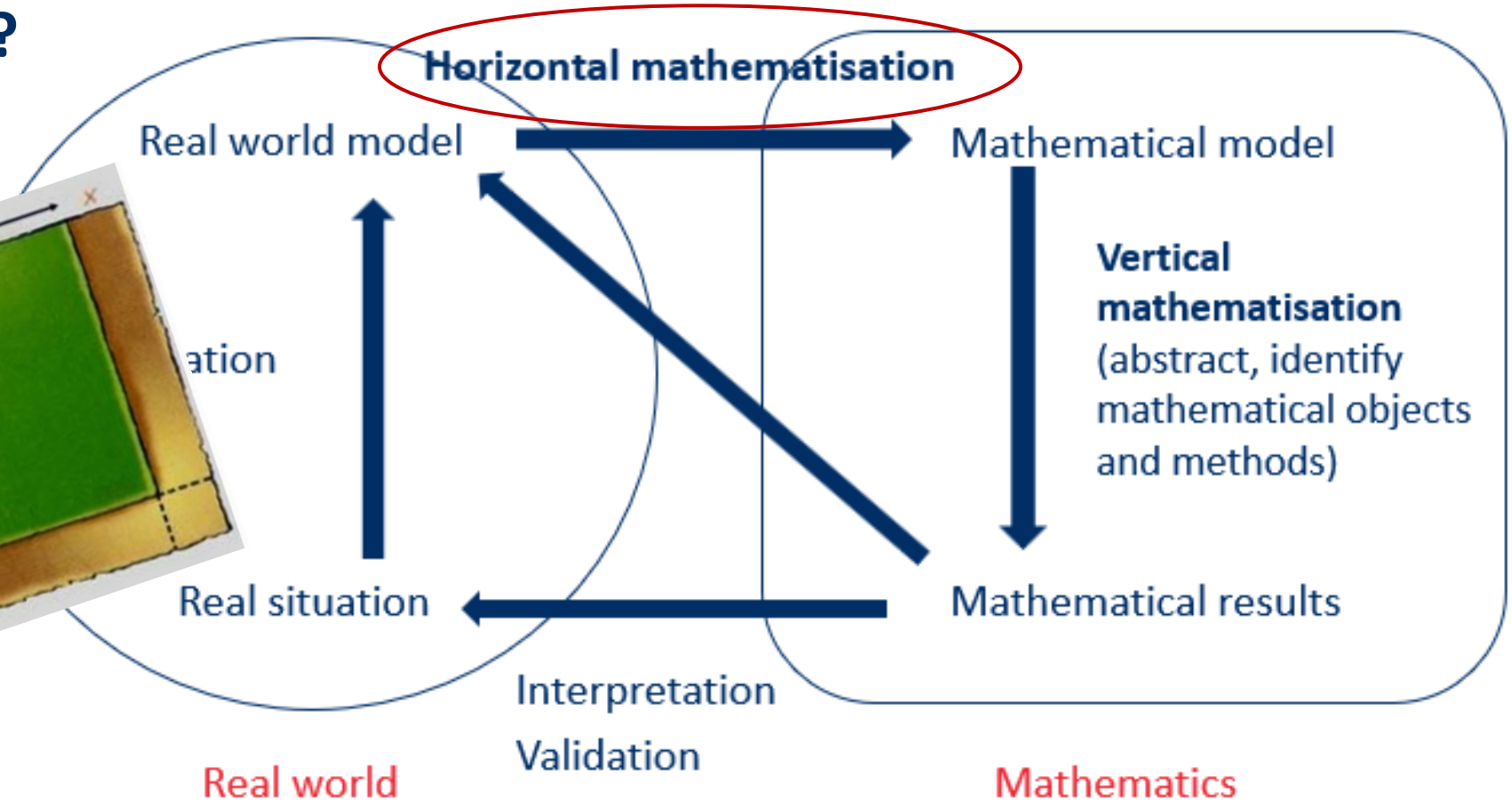
- **Students go through all steps of the modeling cycle (G. Kaiser, 1995)**



Analyze the thinking on assignment C (packed).

Which aspect of the modeling cycle (G. Kaiser, 1995) is not included in exercise C?





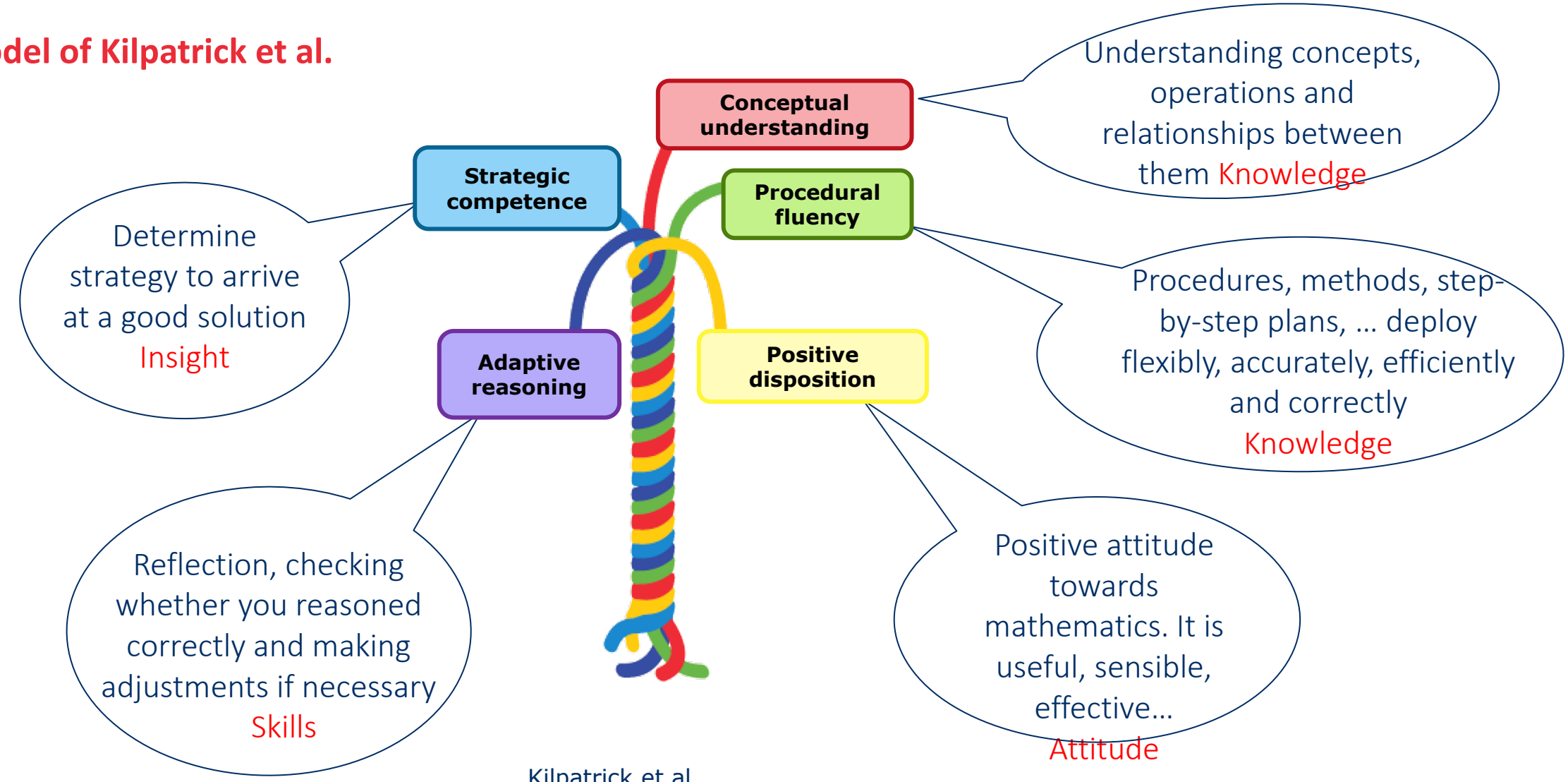
Conclusion

- Pay attention to exercises in which both horizontal and vertical mathematization take place

- This benefits the mathematical proficiency of students

Flashback day 1: mathematical proficiency

Model of Kilpatrick et al.



Kilpatrick et al.

Exercise C and Kilpatrick's model

The lawn in Mr. Jones's garden measures 15 by 20 meters. Mr. Jones decides to extend the lawn. To two sides he adds a strip of equal width of x meters. See Figure 7.16.

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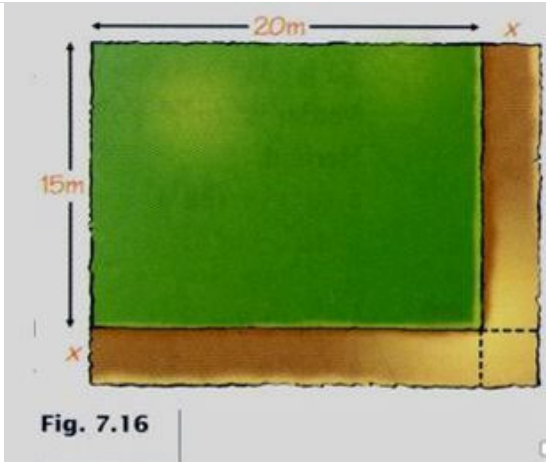
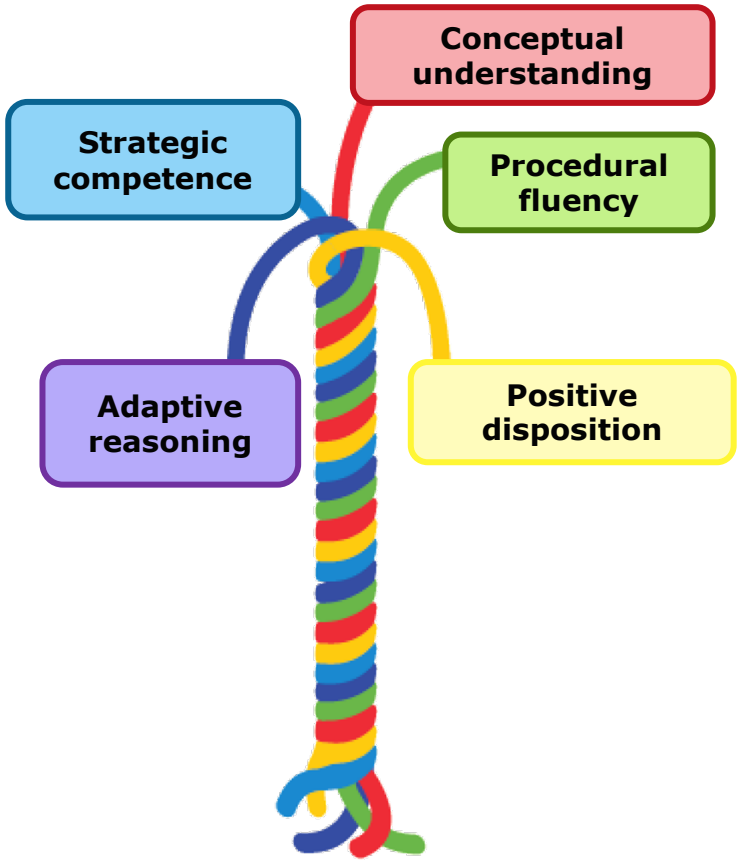


Fig. 7.16



- Mainly procedural fluency
- Strategic competence, adaptive reasoning and positive disposition to a lesser extent

Exercise D and Kilpatrick's model

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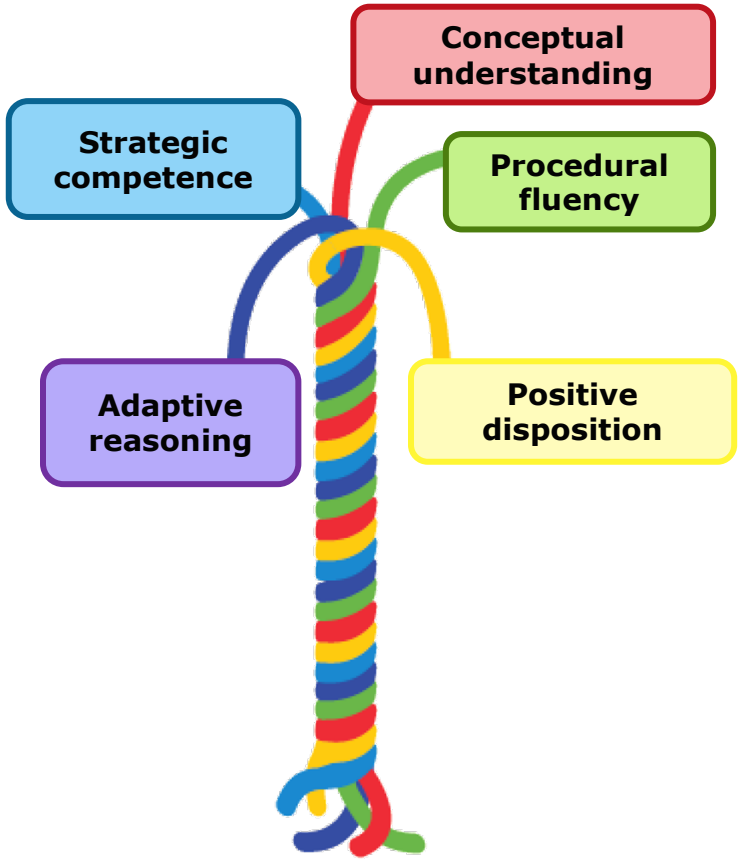

- Relies much more on strategic competence, adaptive reasoning, conceptual understanding and positive disposition than exercise C

Questions?

Assignment

- Have a look at the English mathematics textbooks
- Search for authentic examples and exercises
- Identify 'packed' exercises. What could you do to make these exercised more authentic?