R

Realistic Mathematics Education

Marja Van den Heuvel-Panhuizen[1] and
Paul Drijvers[2]
[1]Freudenthal Institute for Science and Mathematics Education, Faculty of Science & Faculty of Social and Behavioural Sciences, Utrecht University, Utrecht, The Netherlands
[2]Freudenthal Institute, Utrecht University, Utrecht, The Netherlands

Keywords

Domain-specific teaching theory; Realistic contexts; Mathematics as a human activity; Mathematization

What is Realistic Mathematics Education?

Realistic Mathematics Education – hereafter abbreviated as RME – is a domain-specific instruction theory for mathematics, which has been developed in the Netherlands. Characteristic of RME is that rich, "realistic" situations are given a prominent position in the learning process. These situations serve as a source for initiating the development of mathematical concepts, tools, and procedures and as a context in which students can in a later stage apply their mathematical knowledge, which then gradually has become more formal and general and less context specific.

Although "realistic" situations in the meaning of "real-world" situations are important in RME, "realistic" has a broader connotation here. It means students are offered problem situations which they can imagine. This interpretation of "realistic" traces back to the Dutch expression "zich REALISEren," meaning "to imagine." It is this emphasis on making something real in your mind that gave RME its name. Therefore, in RME, problems presented to students can come from the real world but also from the fantasy world of fairy tales, or the formal world of mathematics, as long as the problems are experientially real in the student's mind.

The Onset of RME

The initial start of RME was the founding in 1968 of the Wiskobas ("mathematics in primary school") project initiated by Edu Wijdeveld and Fred Goffree and joined not long after by Adri Treffers. In fact, these three mathematics didacticians created the basis for RME. In 1971, when the Wiskobas project became part of the newly established IOWO Institute, with Hans Freudenthal as its first director and in 1973 when the IOWO was expanded with the Wiskivon project for secondary mathematics education; this basis received a decisive impulse to reform the prevailing approach to mathematics education.

In the 1960s, mathematics education in the Netherlands was dominated by a mechanistic teaching approach; mathematics was taught

S. Lerman (ed.), *Encyclopedia of Mathematics Education*, DOI 10.1007/978-94-007-4978-8,

directly at a formal level, in an atomized manner, and the mathematical content was derived from the structure of mathematics as a scientific discipline. Students learned procedures step by step with the teacher demonstrating how to solve problems. This led to inflexible and reproduction-based knowledge. As an alternative for this mechanistic approach, the "New Math" movement deemed to flood the Netherlands. Although Freudenthal was a strong proponent of the modernization of mathematics education, it was his merit that Dutch mathematics education was not affected by the formal approach of the New Math movement and that RME could be developed.

Freudenthal's Guiding Ideas About Mathematics and Mathematics Education

Hans Freudenthal (1905–1990) was a mathematician born in Germany who in 1946 became a professor of pure and applied mathematics and the foundations of mathematics at Utrecht University in the Netherlands. As a mathematician he made substantial contributions to the domains of geometry and topology.

Later in his career, Freudenthal (1968, 1973, 1991) became interested in mathematics education and argued for teaching mathematics that is relevant for students and carrying out thought experiments to investigate how students can be offered opportunities for guided re-invention of mathematics.

In addition to empirical sources such as textbooks, discussions with teachers, and observations of children, Freudenthal (1983) introduced the method of the didactical phenomenology. By describing mathematical concepts, structures, and ideas in their relation to the phenomena for which they were created, while taking into account students' learning process, he came to theoretical reflections on the constitution of mental mathematical objects and contributed in this way to the development of the RME theory.

Freudenthal (1973) characterized the then dominant approach to mathematics education in which scientifically structured curricula were used and students were confronted with ready-made mathematics as an "anti-didactic inversion." Instead, rather than being receivers of ready-made mathematics, students should be active participants in the educational process, developing mathematical tools and insights by themselves. Freudenthal considered mathematics as a human activity. Therefore, according to him, mathematics should not be learned as a closed system but rather as an activity of mathematizing reality and if possible even that of mathematizing mathematics.

Later, Freudenthal (1991) took over Treffers' (1987a) distinction of horizontal and vertical mathematization. In horizontal mathematization, the students use mathematical tools to organize and solve problems situated in real-life situations. It involves going from the world of life into that of symbols. Vertical mathematization refers to the process of reorganization within the mathematical system resulting in shortcuts by using connections between concepts and strategies. It concerns moving within the abstract world of symbols. The two forms of mathematization are closely related and are considered of equal value. Just stressing RME's "real-world" perspective too much may lead to neglecting vertical mathematization.

The Core Teaching Principles of RME

RME is undeniably a product of its time and cannot be isolated from the worldwide reform movement in mathematics education that occurred in the last decades. Therefore, RME has much in common with current approaches to mathematics education in other countries. Nevertheless, RME involves a number of core principles for teaching mathematics which are inalienably connected to RME. Most of these core teaching principles were articulated originally by Treffers (1978) but were reformulated over the years, including by Treffers himself.

In total six principles can be distinguished:

- The *activity principle* means that in RME students are treated as active participants in the learning process. It also emphasizes that mathematics is best learned by doing mathematics, which is strongly reflected in Freudenthal's interpretation of mathematics as a human activity, as well as in Freudenthal's and Treffers' idea of mathematization.

- The *reality principle* can be recognized in RME in two ways. First, it expresses the importance that is attached to the goal of mathematics education including students' ability to apply mathematics in solving "real-life" problems. Second, it means that mathematics education should start from problem situations that are meaningful to students, which offers them opportunities to attach meaning to the mathematical constructs they develop while solving problems. Rather than beginning with teaching abstractions or definitions to be applied later, in RME, teaching starts with problems in rich contexts that require mathematical organization or, in other words, can be mathematized and put students on the track of informal context-related solution strategies as a first step in the learning process.
- The *level principle* underlines that learning mathematics means students pass various levels of understanding: from informal context-related solutions, through creating various levels of shortcuts and schematizations, to acquiring insight into how concepts and strategies are related. Models are important for bridging the gap between the informal, context-related mathematics and the more formal mathematics. To fulfill this bridging function, models have to shift – what Streefland (1985, 1993, 1996) called – from a "model of" a particular situation to a "model for" all kinds of other, but equivalent, situations (see also Gravemeijer 1994; Van den Heuvel-Panhuizen 2003).

  Particularly for teaching operating with numbers, this level principle is reflected in the didactical method of "progressive schematization" as it was suggested by Treffers (1987b) and in which transparent whole-number methods of calculation gradually evolve into digit-based algorithms.
- The *intertwinement principle* means mathematical content domains such as number, geometry, measurement, and data handling are not considered as isolated curriculum chapters but as heavily integrated. Students are offered rich problems in which they can use various mathematical tools and knowledge. This principle also applies within domains. For example, within the domain of number sense, mental arithmetic, estimation, and algorithms are taught in close connection to each other.
- The *interactivity principle* of RME signifies that learning mathematics is not only an individual activity but also a social activity. Therefore, RME favors whole-class discussions and group work which offer students opportunities to share their strategies and inventions with others. In this way students can get ideas for improving their strategies. Moreover, interaction evokes reflection, which enables students to reach a higher level of understanding.
- The *guidance principle* refers to Freudenthal's idea of "guided re-invention" of mathematics. It implies that in RME teachers should have a proactive role in students' learning and that educational programs should contain scenarios which have the potential to work as a lever to reach shifts in students' understanding. To realize this, the teaching and the programs should be based on coherent long-term teaching-learning trajectories.

Various Local Instruction Theories

Based on these general core teaching principles, a number of local instruction theories and paradigmatic teaching sequences focusing on specific mathematical topics have been developed over time. Without being exhaustive some of these local theories are mentioned here. For example, Van den Brink (1989) worked out new approaches to addition and subtraction up to 20. Streefland (1991) developed a prototype for teaching fractions intertwined with ratios and proportions. De Lange (1987) designed a new approach to teaching matrices and discrete calculus. In the last decade, the development of local instruction theories was mostly integrated with the use of digital technology as investigated by Drijvers (2003) with respect to promoting students' understanding of algebraic concepts and operations. Similarly, Bakker (2004) and Doorman (2005) used dynamic computer software to contribute to an empirically grounded instruction theory for early statistics education and for differential calculus in connection with kinematics, respectively.

The basis for arriving at these local instruction theories was formed by design research, as elaborated by Gravemeijer (1994), involving a theory-guided cyclic process of thought experiments, designing a teaching sequence, and testing it in a teaching experiment, followed by a retrospective analysis which can lead to necessary adjustments of the design.

Last but not least, RME also led to new approaches to assessment in mathematics education (De Lange 1987, 1995; Van den Heuvel-Panhuizen 1996).

Implementation and Impact

In the Netherlands, RME had and still has a considerable impact on mathematics education. In the 1980s, the market share of primary education textbooks with a traditional, mechanistic approach was 95 % and the textbooks with a reform-oriented approach – based on the idea of learning mathematics in context to encourage insight and understanding – had a market share of only 5 %. In 2004, reform-oriented textbooks reached a 100 % market share and mechanistic ones disappeared. The implementation of RME was guided by the RME-based curriculum documents including the so-called Proeve publications by Treffers and his colleagues, which were published from the late 1980s, and the TAL teaching-learning trajectories for primary school mathematics, which have been developed from the late 1990s (Van den Heuvel-Panhuizen 2008; Van den Heuvel-Panhuizen and Buys 2008).

A similar development can be seen in secondary education, where the RME approach also influenced textbook series to a large extent. For example, Kindt (2010) showed how practicing algebraic skills can go beyond repetition and be thought provoking. Goddijn et al. (2004) provided rich resources for realistic geometry education, in which application and proof go hand in hand.

Worldwide, RME is also influential. For example, the RME-based textbook series "Mathematics in Context" Wisconsin Center for Education Research & Freudenthal Institute (2006) has a considerable market share in the USA. A second example is the RME-based "Pendidikan Matematika Realistik Indonesia" in Indonesia (Sembiring et al. 2008).

A Long-Term and Ongoing Process of Development

Although it is now some 40 years from the inception of the development of RME as a domain-specific instruction theory, RME can still be seen as work in progress. It is never considered a fixed and finished theory of mathematics education. Moreover, it is also not a unified approach to mathematics education. That means that through the years different emphasis was put on different aspects of this approach and that people who were involved in the development of RME – mostly researchers and developers of mathematics education and mathematics educators from within or outside the Freudenthal Institute – put various accents in RME. This diversity, however, was never seen as a barrier for the development of RME but rather as stimulating reflection and revision and so supporting the maturation of the RME theory. This also applies to the current debate in the Netherlands (see Van den Heuvel-Panhuizen 2010) which voices the return to the mechanistic approach of four decades back. Of course, going back in time is not a "realistic" option, but this debate has made the proponents of RME more alert to keep deep understanding and basic skills more in balance in future developments of RME and to enhance the methodological robustness of the research that accompanies the development of RME.

Cross-References

▶ Didactical Phenomenology (Freudenthal)

References

Bakker A (2004) Design research in statistics education: on symbolizing and computer tools. CD-Bèta Press, Utrecht

De Lange J (1987) Mathematics, insight and meaning. OW & OC, Utrecht University, Utrecht

De Lange J (1995) Assessment: no change without problems. In: Romberg TA (ed) Reform in school mathematics. SUNY Press, Albany, pp 87–172

Doorman LM (2005) Modelling motion: from trace graphs to instantaneous change. CD-Bèta Press, Utrecht

Drijvers P (2003) Learning algebra in a computer algebra environment. Design research on the understanding of the concept of parameter. CD-Bèta Press, Utrecht

Freudenthal H (1968) Why to teach mathematics so as to be useful. Educ Stud Math 1:3–8

Freudenthal H (1973) Mathematics as an educational task. Reidel Publishing, Dordrecht

Freudenthal H (1983) Didactical phenomenology of mathematical structures. Reidel Publishing, Dordrecht

Freudenthal H (1991) Revisiting mathematics education. China lectures. Kluwer, Dordrecht

Goddijn A, Kindt M, Reuter W, Dullens D (2004) Geometry with applications and proofs. Freudenthal Institute, Utrecht

Gravemeijer KPE (1994) Developing realistic mathematics education. CD-ß Press/Freudenthal Institute, Utrecht

Kindt M (2010) Positive algebra. Freudenthal Institute, Utrecht

Sembiring RK, Hadi S, Dolk M (2008) Reforming mathematics learning in Indonesian classrooms through RME. ZDM Int J Math Educ 40(6):927–939

Streefland L (1985) Wiskunde als activiteit en de realiteit als bron. Nieuwe Wiskrant 5(1):60–67

Streefland L (1991) Fractions in realistic mathematics education. A paradigm of developmental research. Kluwer, Dordrecht

Streefland L (1993) The design of a mathematics course. A theoretical reflection. Educ Stud Math 25(1–2):109–135

Streefland L (1996) Learning from history for teaching in the future. Regular lecture held at the ICME-8 in Sevilla, Spain; in 2003 posthumously. Educ Stud Math 54:37–62

Treffers A (1978) Wiskobas doelgericht [Wiskobas goal-directed]. IOWO, Utrecht

Treffers A (1987a) Three dimensions. A model of goal and theory description in mathematics instruction – the Wiskobas project. D. Reidel Publishing, Dordrecht

Treffers A (1987b) Integrated column arithmetic according to progressive schematisation. Educ Stud Math 18:125–145

Van den Brink FJ (1989) Realistisch rekenonderwijs aan jonge kinderen [Realistic mathematics education for young children]. OW&OC, Universiteit Utrecht, Utrecht

Van den Heuvel-Panhuizen M (1996) Assessment and realistic mathematics education. CD-ß Press/Freudenthal Institute, Utrecht University, Utrecht

Van den Heuvel-Panhuizen M (2003) The didactical use of models in realistic mathematics education: an example from a longitudinal trajectory on percentage. Educ Stud Math 54(1):9–35

Van den Heuvel-Panhuizen M (ed) (2008) Children learn mathematics. A learning-teaching trajectory with intermediate attainment targets for calculation with whole numbers in primary school. Sense Publishers, Rotterdam/Tapei

Van den Heuvel-Panhuizen M (2010) Reform under attack – forty years of working on better mathematics education thrown on the scrapheap? no way! In: Sparrow L, Kissane B, Hurst C (eds) Shaping the future of mathematics education: proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia. MERGA, Fremantle, pp 1–25

Van den Heuvel-Panhuizen M, Buys K (eds) (2008) Young children learn measurement and geometry. Sense Publishers, Rotterdam/Taipei

Wisconsin Center for Education Research & Freudenthal Institute (ed) (2006) Mathematics in context. Encyclopaedia Britannica, Chicago

Recontextualization in Mathematics Education

Paul Dowling
Institute of Education, Department of Culture, Communication and Media, University of London, London, UK

Keywords

Anthropological theory of didactics; Classification; Didactic transposition; Discursive saturation; Domains of action; Emergence; Framing; Institutionalisation; Noosphere; Pedagogic device; Recontextualisation; Social activity method; Sociology; Strategic action

Characteristics

Recontextualization refers to the contention that texts and practices are transformed as they are moved between contexts of their reading or enactment. This simple claim has profound implications for mathematics education and for education generally. There are three major theories in the general field of educational studies that directly and explicitly concern recontextualization: the Theory of Didactic Transposition (later the Anthropological Theory of Didactics) of Yves Chevallard, Basil Bernstein's pedagogic device, and Paul Dowling's Social Activity Method. These are all complex theories, so their presentation here of necessity entails substantial simplification.

The Theory of Didactic Transposition (TDT) proposes, essentially, that constituting a practice as something to be taught will always involve a transformation of the practice. This is a general claim that can be applied to any practice and any form of teaching, but Chevallard's (1985, 1989) work and that of many of those who have worked with the TDT is most centrally concerned with the teaching of mathematics in formal schooling (primary, secondary, or higher education phases). The work of the didactic transposition is carried out, firstly, by agents of what Chevallard referred to as the noosphere and involves the production of curricula in the form of policy documents, syllabuses, textbooks, examinations, and so forth constituting the "knowledge to be taught." The first task in this work is the construction of a body of source knowledge as the referent practice of the "knowledge to be taught." In the case of school mathematics, this source or "scholarly knowledge" has been produced by mathematicians over a very long historical period and in diverse contexts. In its totality, then, it is not a practice that is currently enacted by mathematicians, but is compiled in the noosphere. The next task is the constitution of the "knowledge to be taught" from this "scholarly knowledge," and it is the former that is presented to teachers as the curriculum. There is a further move, however, as the teacher in the classroom must, through interpretation and the production and management of lessons, transpose the "knowledge to be taught" into "knowledge actually taught." Even this knowledge is not necessarily equivalent to the knowledge acquired by the student, which is the product of a further transposition. The precise nature of the transposition at each stage is a function of the nature of the knowledge (scholarly, to be taught, actually taught) being recontextualized and of historical, cultural, and pedagogic specificities. TDT – which has been developed in terms of conceptual complexity as the Anthropological Theory of Didactics (ATD, Chevallard 1992) – invites researchers to investigate the precise processes whereby the recontextualizations have been achieved in particular locations and in respect of particular regions of the curriculum, so revealing the conditions and constraints on the teaching of mathematics in these contexts. This has been attempted in, for example, the topics of calculus (Bergsten et al. 2010), statistics (Wozniak 2007), and the limits of functions (Barbé et al. 2005).

Bernstein describes the "pedagogic device" as "the condition for culture, its productions, reproductions and the modalities of their interrelations" (1990; see also Bernstein 2000). It is a central feature of a highly complex theory that was developed over a period of some 40 years, so its representation here is of necessity radically simplified. Whereas Chevallard's theory is concerned with the epistemological and cultural constraints on didactics, Bernstein's interest lies in the manner in which societies are reproduced and transformed. Pedagogy and, in particular, transmission occur in all sociocultural institutions, although much of the work inspired by Bernstein has focused on formal schooling. An important exception to this is his early dialogue with the anthropologist, Mary Douglas (see Douglas 1996/1970), which contributed to Douglas's cultural theory and Bernstein's fundamental concepts, *classification* (regulation between contexts) and framing (regulation within a context). The *pedagogic device* regulates what is transmitted to whom, when, and how and consists of three sets of rules, hierarchically organized: *distribution*, *recontextualization*, and *evaluation*. Recontextualization rules, in particular, regulate the delocation of discourses from the fields of their production – the production of physics discourse in the university, for example – and their relocation as pedagogic discourse. This is achieved by the embedding of these *instructional discourses* in *regulatory discourses* involving principles of selection, sequencing, and pacing. Recontextualization is achieved by agents in the *official recontextualizing field* – policy makers and administrators – and the *pedagogic recontextualizing field* (teacher educators, the authors of textbooks, and so forth) that together might be taken to coincide with Chevallard's noosphere in terms of membership. Superficially, there might seem to be similarities between Bernstein's and Chevallard's theories. A crucial distinction, however, is that recontextualization for Bernstein, but not for Chevallard, is always governed by distribution. This entails that pedagogic discourse is always structured by the social dimensions of class, gender, and race. Bernstein's is a sociological theory, while Chevallard's might reasonably be described (in English) as an educational theory. Through the sociological concept, relative autonomy, Bernstein also allows for the possibility of the transformation of culture and, ultimately, of society. A further distinction lies in that Bernstein describes pedagogic discourse in terms of his fundamental categories, classification, and framing, which enables a description of form but not of content. Further resources for the description of the form of discourses are available in Bernstein's (2000) work on *horizontal* and *vertical discourses* and on *knowledge structures* where he describes mathematics as a vertical discourse having horizontal knowledge structure and a strong grammar. In this description he seems to be making no epistemological distinction between mathematics in its field of production, on the one hand, and school mathematics, on the other.

Dowling's (2009, 2013) *Social Activity Method* (SAM) presents a sociological organizational language that takes seriously lessons from constructionism and poststructuralism. As is the case with Chevallard's TDT, Dowling's work began with an interest in mathematics education (see Dowling 1994, 1995, 1996, 1998) but is more fundamentally sociological, giving a degree of priority to social relations over *cultural practices*. For Dowling, the sociocultural is characterized by *social action* that is directed at the formation, maintenance, and destabilizing of *alliances* and *oppositions*. These alliances and oppositions, however, are emergent upon the totality of social action rather than being the deliberate outcomes of individual actions. Alliances are visible in terms of regularities of practice that give the appearance of regulating who can do, say, think what, though, as emergent outcomes, they might be thought of, metaphorically, as advisory rather than determinant. Another feature of Dowling's theory is that it has a fractal quality, which is to say, the same language can be applied at any level of analysis and the language is also capable of describing itself. School mathematics is an example of what might be taken to exhibit a regularity of practice including the institutionalization of expression (signifiers) and content (signifieds) in texts. The strength of institutionalization varies, however, between strong and weak, giving rise to the scheme of *domains of practice* in Fig. 1, which constitutes part of the structure of all contexts, which is to say, of all alliances. Human agents might be described as seeing the world in terms of the scheme in Fig. 1 or, more precisely, from the perspective of the *esoteric domain*. Where the particular context is school mathematics, the agent may cast a *gaze* beyond school mathematics onto, for example, domestic practices such as

	Content (signifieds)	
Expression (signifiers)	I^+	I^-
I^+	*esoteric domain*	*descriptive domain*
I^-	*expressive domain*	*public domain*

$I^{+/-}$ represents strong/weak institutionalisation.

Recontextualization in Mathematics Education, Fig. 1 Domains of action (Source: Dowling 2009)

Recontextualization in Mathematics Education, Fig. 2 Modes of recontextualization (Source: Dowling 2013)

Representation	Practice	
	DS^-	DS^+
DS^-	*improvising*	*de-principling*
DS^+	*rationalising*	*re-principling*

shopping. The deployment of principles of recognition and realization that are specific to school mathematics will result in the recontextualization of domestic shopping as mathematized shopping. This contributes to the *public domain* of school mathematics, which thereby appears to be about something other than mathematics. This contrasts with *esoteric domain* text that is unambiguously about mathematics, the *descriptive domain* – the domain of mathematical modelling – that appears to be about something other than mathematics but that is presented in the language of mathematics, and the *expressive domain* (the domain of pedagogic metaphors) that appears to be about mathematics but that is presented in the language of other practices (an equation is a balance, and so forth). This scheme enables the description of complex mathematical texts and settings in terms of the distribution of the different domains of mathematical practice to different categories of student (e.g., in terms of social class). It can also reveal distinctions between modes of pedagogy that take different trajectories around the scheme. It should be emphasized that *public domain* shopping is not the same thing as domestic shopping; the recontextualization of practice always entails a transformation as is illustrated by Brantlinger (2011) in respect of critical mathematics education. The *gaze* of mathematics education is described (Dowling 2010) as *fetching* practices from other activities and recontextualizing them as mathematical practice. This is, in a sense, a didactic necessity in the production of apprentices to mathematics who must, initially, be addressed in a language that is familiar to them. A danger, however, lies in the *pushing* of the results back out of mathematics as the result no longer has ecological validity. The scheme in Fig. 1 is reproduced in all activities that can be recognized as exhibiting regularity of practice and at all levels within any such practice. Chung (2011), for example, has directed an elaborated version of the scheme at literary studies.

Another category from SAM is *discursive saturation*, which refers to the extent to which a practice makes its principles linguistically available. To the extent that an activity or part of an activity can be described as high or low discursive saturation (DS^+ or DS^-), then another scheme is generated that describes modes of recontextualization. This scheme is shown in Fig. 2. If school mathematics can generally be described as DS^+ and domestic shopping as DS^-, then the recontextualizing of domestic shopping as school mathematics public domain – the representation of shopping by mathematics – can be described as *rationalizing* and the recontextualizing of, say, banking by school mathematics as *re-principling*.

These three theories of *recontextualization* – those of Chevallard, Bernstein, and Dowling – offer different possibilities to researchers, and practitioners in mathematics education and themselves draw on different theoretical and disciplinary antecedents. They are, however, not in competition as much as being complementary. All three present languages that can be and have been deployed far more widely than mathematics education, though Chevallard's and Dowling's theories certainly have their roots in this field of research. Naturally, all three theories have undergone more or less transformative action in respect of their recontextualization for the purposes of this entry.

Cross-References

▶ Anthropological Approaches in Mathematics Education, French Perspectives
▶ Calculus Teaching and Learning
▶ Critical Mathematics Education

- ▶ Curriculum Resources and Textbooks in Mathematics Education
- ▶ Didactic Transposition in Mathematics Education
- ▶ Gender in Mathematics Education
- ▶ Language Background in Mathematics Education
- ▶ Mathematical Knowledge for Teaching
- ▶ Mathematical Language
- ▶ Mathematics Curriculum Evaluation
- ▶ Metaphors in Mathematics Education
- ▶ Semiotics in Mathematics Education
- ▶ Socioeconomic Class in Mathematics Education
- ▶ Sociological Approaches in Mathematics Education

References

Barbé J, Bosch M, Espinoza L, Gascón J (2005) Didactic restrictions on the teacher's practice: the case of limits of functions in Spanish high schools. Educ Stud Math 59:235–268

Bergsten C, Jablonka E, Klisinska A (2010) A remark on didactic transposition theory. In: Bergsten C, Jablonka E, Wedege T (eds) Mathematics and mathematics education: cultural and social dimensions. Proceedings of MADIF7. SMDF, Linköping, pp 58–68

Bernstein B (1990) Class, codes and control, vol IV, The structuring of pedagogic discourse. RKP, London

Bernstein B (2000) Pedagogy, symbolic control and identity. Rowman & Littlefield, New York

Brantlinger A (2011) Rethinking critical mathematics: a comparative analysis of critical, reform, and traditional geometry instructional texts. Educ Stud Math 78:395–411

Chevallard Y (1985) La transposition didactique du savoir savant au savoir enseigné. Editions Pensée Sauvage, Grenoble

Chevallard Y (1989) On didactic transposition theory: some introductory notes. In: The proceedings of the international symposium on selected domains of research and development in mathematics education, Bratislava, 3–7 August 1988, pp 51–62. http://yves.chevallard.free.fr/spip/spip/IMG/pdf/On_Didactic_Transposition_Theory.pdf. Accessed 26 Aug 2012

Chevallard Y (1992) Fundamental concepts in didactics: perspectives provided by an anthropological approach. In: Douady R, Mercier A (eds) Research in didactique of mathematics. La Pensée Sauvage, Grenoble, pp 131–167

Chung S (2011) The modality of the textual institutionalisation of literary studies: towards a sociology. Sociol Res Online 16(3/3):1–14

Douglas M (1996/1970) Natural symbols: explorations in cosmology. Routledge, London

Dowling PC (1994) Discursive saturation and school mathematics texts: a strand from a language of description. In: Ernest P (ed) Mathematics, education and philosophy: an international perspective. Falmer Press, London, pp 124–142

Dowling PC (1995) A language for the sociological description of pedagogic texts with particular reference to the secondary school mathematics scheme *SMP 11–16*. Collect Orig Resour Educ 19(2): no journal page numbers

Dowling PC (1996) A sociological analysis of school mathematics texts. Educ Stud Math 31:389–415

Dowling PC (1998) The sociology of mathematics education: mathematical myths/pedagogic texts. Falmer Press, London

Dowling PC (2009) Sociology as method: departures from the forensics of culture, text and knowledge. Sense, Rotterdam

Dowling PC (2010) Abandoning mathematics and hard labour in schools: a new sociology of knowledge and curriculum reform. In: Bergsten C, Jablonka E, Wedege T (eds) Mathematics and mathematics education: cultural and social dimensions. Proceedings of MADIF7. SMDF, Linköping, pp 1–30

Dowling PC (2013) Social activity method: a fractal language for mathematics. Math Educ Res J 25:317–340

Wozniak F (2007) Conditions and constraints in the teaching of statistics: the scale of levels of determination. In: Congress of the European Society for Research in Mathematics Education, 5. Larnaca, 22–26 February 2007. http://ermeweb.free.fr/CERME%205/WG11/11_Wozniak.pdf Accessed 26 Aug 2012

Reflective Practitioner in Mathematics Education

Barbara Jaworski
Loughborough University, Loughborough, Leicestershire, UK

Reflective practice is a commonly used term in mathematics education, often without careful definition, implying a contemplative reviewing of learning and/or teaching in mathematics in order to approve, evaluate, or improve practice. A feedback loop is often suggested in which reflective practice feeds back into the design or initiation of practice providing possibilities for improved practice. More precise definitions often draw on Dewey, who wrote:

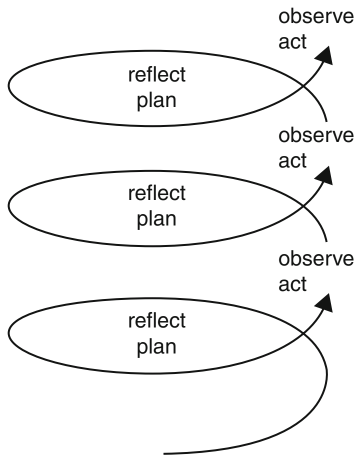
> Active, persistent and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it and the further conclusions to which it tends constitutes reflective thought (1933, p. 9)
>
> . . . reflective thinking, in distinction to other operations to which we apply the name of thought, involves (1) a state of doubt, hesitation, perplexity, mental difficulty, in which thinking originates, and (2) an act of searching, hunting, inquiring, to find material that will resolve the doubt and dispose of the perplexity (p. 12).
>
> . . . Demand for the solution of a perplexity is the steadying and guiding factor in the entire process of reflection. (p. 14)

Rather than a perspective just of contemplative thought, Dewey emphasizes the important element of *action* in reflection and the goal of an *action outcome*. This has led to a linking of reflective practice with so-called *action research* which is research conducted by practitioners into aspects of (their own) professional practice. Stephen Kemmis a leading proponent of action research spoke of reflection as "meta-thinking," thinking about thinking. He wrote:

> We do not pause to reflect in a vacuum. We pause to reflect because some issue arises which demands that we stop and take stock or consider before we act. . . . We are inclined to see reflection as something quiet and personal. My argument here is that reflection is action-oriented, social and political. Its product is praxis (informed committed action) the most eloquent and socially significant form of human action. (Kemmis 1985, p. 141)

Kemmis conceptualized action research with reference to a critically reflective spiral in action research of plan, act and observe, and reflect (Kemmis and McTaggart 1981; Carr and Kemmis 1986), and other scholars have adapted this subsequently (e.g., McNiff 1988) (Fig. 1).

More recent scholars relate ideas of reflection, seminally, to the work of Donald Schön who has written about the *reflective practitioner* in professions generally and in education particularly (Schön 1983, 1987). Schön relates reflection to *knowing* and describes *knowing-in-action* and *reflection-in-action*. With reference to Dewey, he writes about *learning by doing*, the importance



Reflective Practitioner in Mathematics Education, Fig. 1 Action-reflection cycle (McNiff (1988), pp 44, Fig 3.7)

of action in the process of learning, and relates *doing* and *learning* through a reflective process.

> Our knowing is ordinarily tacit, implicit in our patterns of action and in our feel for the stuff with which we are dealing. It seems right to say that our knowing is in our action (1983, p. 49).

Schön refers to *knowing-in-action* as "the sorts of know-how we reveal in our intelligent action – publicly observable, physical performances like riding a bicycle and private operations like instant analysis of a balance sheet" (1997, p. 25). He claims a subtle distinction between knowing-in-action and reflection-in-action. The latter he links to moments of *surprise* in action: "We may reflect *on* action, thinking back on what we have done in order to discover how our knowing-in-action may have contributed to an unexpected outcome" (p. 26). "Alternatively," he says, "we may reflect in the midst of action without interrupting it . . . our thinking serves to reshape what we are doing while we are doing it" (p. 26). Schön distinguishes reflection-on-action and reflection-in-action. The first involves looking back *on* an action and reviewing its provenance and outcomes with the possibility then of modifying future action; the second is especially powerful, allowing the person acting to recognize a moment in the action, possibly with surprise, and to act, there and then, differently. John Mason has taken up this idea in his *discipline of noticing*: we *notice*, in the moment, something of which we are aware, possibly have reflected *on* in the past and our noticing afford us the opportunity to act differently, to modify our actions in the process of acting (Mason 2002).

Michael Eraut (1995) has criticized Schön's theory of reflection-in-action where it applies to teachers in classrooms. He points out that Schön presents little empirical evidence of reflection-in-action, especially where teaching is concerned. The word *action* itself has different meanings for different professions. In teaching, *action* usually refers to action in the classroom where teachers operate under pressure. Eraut argues that time constraints in teaching limit the scope for reflection-in-action. He argues that there is too little time for considered reflection as part of the teaching act, especially where teachers are responding to or interacting with students. Where a teacher is walking around a classroom of children quietly working on their own, reflection-in-action is more possible but already begins to resemble time *out* of action. Thus Eraut suggests that, in teaching, most reflection is reflection-*on*-action, or reflection-*for*-action. He suggests that Schön is primarily concerned with reflection-*for*-action, reflection whose purpose is to affect action in current practice.

In mathematics education research into teaching practices in mathematics classrooms, Jaworski (1998) has worked with the theoretical ideas of Schön, Mason, and Eraut to characterize observed mathematics teaching and the thinking, action, and development of the observed teachers. The research was undertaken as part of a project, the Mathematics Teachers' Enquiry (MTE) Project, in which participating teachers engaged in forms of action research into their own teaching. Jaworski claims that the three prepositions highlighted in the above discussion, *on*, *in*, and *for*, "all pertain to the thinking of teachers at different points in their research" (p. 9) and provides examples from observations of teaching and conversations with teachers. To some degree, all the teachers observed engaged in action research in the sense that they explored aspects of their own practice in reflective cycles. However, rather than the theorized *systematicity* of action research (e.g., McNiff 1988), Jaworski described the cyclic process of growth of knowledge for these teachers as *evolutionary*, as "lurching" from time to time, opportunity to opportunity, as teachers grappled with the heavy demands of being a teacher and sought nevertheless to reflect on and in their practice. As Eraut suggested, the nature of teaching in classrooms is demanding and complex for the teacher, as is the ongoing life in a school and the range of tasks a teacher is required to undertake. Teachers' reflection *on* their practice, evidenced by reports at project meetings and observations of teacher educator researchers, led to noticing *in* the moment in classrooms, reflection-*in*-action, and concomitant changes in action resulting from such noticing.

A question that arises in considering reflective practice in mathematics education concerns what difference it makes (to reflective practice) that it is being used in relation to mathematics and to the learning and teaching of mathematics. Although in the mathematics education literature there are many references to the reflection of practitioners, there is a singular lack of relating reflective practice directly to mathematics. We see writings by mathematics educators referring, for example, to mathematics teachers who are reflective practitioners, reflecting on their practice of teaching mathematics; however, the mathematics is rarely addressed per se. We read about specific approaches to teaching mathematics and to engagement in reflective practice, for example, the identification of "critical incidents," or the use of a "lesson study approach." To a great extent, the same kinds of practices and issues might be reported if the writers were talking about science or history teaching. There is also a dearth of research in which mathematics students are seen as reflective practitioners.

References

Carr W, Kemmis S (1986) Becoming critical: education, knowledge and action research. Routledge Falmer, London

Dewey J (1933) How we think. D.C. Heath, London

Eraut M (1995) Schön shock: a case for reframing reflection-in-action? Teach Teach Theory Pract 1(1):9–22
Jaworski B (1998) Mathematics teacher research: process, practice and the development of teaching. J Math Teach Educ 1(1):3–31
Kemmis S (1985) Action research and the politics of reflection. In: Boud D, Keogh R, Walker D (eds) Reflection: turning experience into learning. Kogan Page, London
Kemmis S, McTaggart R (1981) The action research planner. Deakin University, Geelong
Mason J (2002) Researching your own classroom practice. Routledge Falmer, London
McNiff J (1988) Action research: principles and practice. Macmillan, London
Schön DA (1983) The reflective practitioner. Temple Smith, London
Schön DA (1987) Educating the reflective practitioner. Jossey-Bass, Oxford

Rural and Remote Mathematics Education

Tom Lowrie
Faculty of Education, Charles Sturt University, Wagga Wagga, NSW, Australia

Keywords

Rural mathematics education; Remote mathematics education; Distance education

Definition(s)

Definitions of rural and remote mathematics contexts differ considerably from country to country and region to region – nevertheless most definitions consider geographical position, population density, and distance from the nearest urban area. The Organisation for Economic Co-operation and Development (OECD) classifies regions within its member countries into three groups based on population density – predominantly urban, intermediate, or predominantly rural. A region is considered rural if it meets three methodology criteria: (1) "local units" within a region are rural if they have a population density of less than 150 inhabitants per square kilometer, (2) more than 50 % of the population in the region live in rural local units, and (3) they will not contain an urban center of over 200,000 people (OECD 2010a).

Developing regions around the world, in particular Africa and Asia, are still mostly rural. However, by 2030 these regions will join the developed world in having a mostly urban population. Although the developed world has been predominantly urban since the early 1950s, some countries have a relative high proportion of the population outside major cities (e.g., Australia, 34 %; Canada, 19 %) (Australian Bureau of Statistics [ABS] 2012; Statistics Canada 2008). Social indicators show that people living in rural areas have less access to a high quality of life than do those living in urban areas, based on factors such as employment, education, health, and leisure (UN 2011). To some degree, research in this area has been considered from a deficit perspective, often perceived as backward, attached to tradition, and anti-modern (Howley et al. 2010).

Differences in Student Performance

Students in large urban areas tend to outperform students in rural schools by the equivalent of more than one year of education (OECD 2012). Severe poverty, often exacerbated in rural areas due to a lack of employment, education opportunities, and infrastructure, manifests the situation (Adler et al. 2009). Although socioeconomic background accounts for part of the difference, performance difference remain even when socioeconomic background is removed as a factor (OECD 2012). In other situations, severe environmental conditions, including drought and flood, heighten the challenging nature of educational opportunities in rural areas (Lowrie 2007). Differences in students' success in mathematics are often correlated with the size of their community, along with its degree of remoteness (Atweh et al. 2012). Rural, and especially remote, communities face challenges of high staff turnover, reduced professional learning opportunities, and difficulty in accessing quality learning opportunities for students (Lyons et al. 2006). The capacity to attract teachers with strong mathematics pedagogical content knowledge – already a challenge in many countries – is heightened in rural and remote areas with students having limited opportunities to study higher levels of mathematics (Kitchenham and Chasteauneuf 2010; Ngo 2012). As the OECD (2010b, p. 13) highlights, "...disadvantaged schools still report great difficulties in attracting qualified teachers... Findings from PISA suggest that, in terms of teacher resources, many students face the double liability of coming from a disadvantaged background and attending a school with lower quality resources."

Opportunities in Rural and Remote Settings

From a pedagogical perspective, communication technologies provide opportunities for enhanced mathematics engagement (Lowrie 2006). In fact, distance education often leads the way in communication initiatives and technological advances (Guri-Rosenblit 2009). A benefit can be that rural/remote schools and students have access to current and innovative technologies that are not yet being used in mainstream metropolitan schools. In this sense, remote settings provide opportunities for mathematics pedagogy to be differently contextualized (Lowrie and Jorgensen 2012).

Distance education features strongly in the organization structuring of education in remote areas – with students afforded the opportunity to study mathematics without leaving their home community. Such situations change the nature and role of teaching – with the student having to be more self-reliant since face-to-face engagement with their teacher is minimal. High-quality teaching and learning are fostered through well-designed resources and strong home-school partnerships (Lowrie 2007). The shared decision-making that is negotiated and established in distance education contexts is highly influential in the students' numeracy development (Goos and Jolly 2004) and can be looked upon in reshaping the practices of more traditional mathematics classrooms.

Cross-References

- ▶ Equity and Access in Mathematics Education
- ▶ Socioeconomic Class in Mathematics Education
- ▶ Urban Mathematics Education

References

Adler J, Pournara C, Taylor D, Thorne B, Moletsane G (2009) Mathematics and science teacher education in South Africa: a review of research, policy and practice in times of change. Afr J Res MST Educ 13(Special Issue): 28–46

Atweh B, Vale C, Walshaw M (2012) Equity, diversity, social justice and ethics in mathematics education. In: Perry B, Lowrie T, Logan T, MacDonald A, Greenlees J (eds) Research in mathematics education in Australasia 2008–2011. Sense, Rotterdam, pp 39–64

Australian Bureau of Statistics [ABS] (2012) 3218.0—Regional population growth, Australia, 2010–11. Author, Canberra, Australia. http://www.abs.gov.au/ausstats/abs@.nsf/Products/3218.0~2010-11~Main+Features~Main+Features?OpenDocument#PARALINK3. Accessed 22 May 2012

Goos M, Jolly L (2004) Building partnership with families and communities to support children's literacy development. In: Putt I, Faragher R, McLean M (eds) Mathematics education for the third millennium: Towards 2010. Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia MERGA. Townsville, Australia, pp. 279–286

Guri-Rosenblit S (2009) Distance education in the digital age: common misconceptions and challenging tasks. J Distance Educ 23(2):105–122

Howley A, Howley C, Klein R, Belcher J, Howley M, Tusay M, Clonch S, Perko H, Foley G, Pendarvis E, Miyafusa S, Jimerson L (2010) Community and place in mathematics instruction in selected rural schools. Appalachian Collaborative Center for Learning, Assessment, and Instruction in Mathematics (ACCLAIM), Ohio University, Athens, Ohio

Kitchenham A, Chasteauneuf C (2010) Teacher supply and demand: issues in Northern Canada. Can J Educ 33:869–896

Lowrie T (2006) Establishing school-family partnerships in distance education contexts: pedagogical engagement in isolated settings. J Distance Educ 21(2):96–114

Lowrie T (2007) Learning engagement in distance and rural settings: four Australian cases. Learn Environ Res 10:35–51

Lowrie T, Jorgensen R (2012) Teaching mathematics remotely: changed practices in distance education. Math Educ Res J 24:371–383

Lyons T, Cooksey R, Panizzon D, Parnell A, Pegg J (2006) Science, ICT and mathematics education in rural and regional Australia: The SiMERR National Survey. A research report prepared for the Department of Education and Training

Ngo FJ (2012) The distribution of pedagogical content knowledge in Cambodia: gaps and thresholds in math achievement. Educational Research for Policy and Practice. doi: 10.1007/s10671-012-9133-1 http://www.springerlink.com/content/x463m5045417x748/ Accessed 1 Aug 2012

Organisation for Economic Co-operation and Development (OECD) (2010a) OECD regional typology. OECD, Paris, France. http://www.oecd.org/dataoecd/35/62/42392595.pdf. Accessed 31 May 2012

Organisation for Economic Co-operation and Development (OECD) (2010b) Refinement of the OECD regional typology: Economic performance of remote rural regions. OECD, Paris, France. http://www.oecd.org/regional/regionaldevelopment/45511797.pdf Accessed 31 May 2012

Organisation for Economic Co-operation and Development (OECD) (2012) PISA in focus: Are large cities educational assets or liabilities? OECD, Paris, France. http://www.pisa.oecd.org/dataoecd/37/8/50611139.pdfAccessed 1 Aug 2012

Statistics Canada (2008) Canadian demographics at a glance. Catalogue no. 91-003-X. Author, Ottawa, Canada. http://www.statcan.gc.ca/pub/91-003-x/91-003-x2007001-eng.pdf Accessed 31 May 2012

United Nations Department of Economic and Social Affairs Population Division (2011) Population distribution, urbanization, internal migration and development: An international perspective. United Nations, Paris, France http://www.un.org/esa/population/publications/PopDistribUrbanization/PopulationDistributionUrbanization.pdf Accessed 22 May 2012