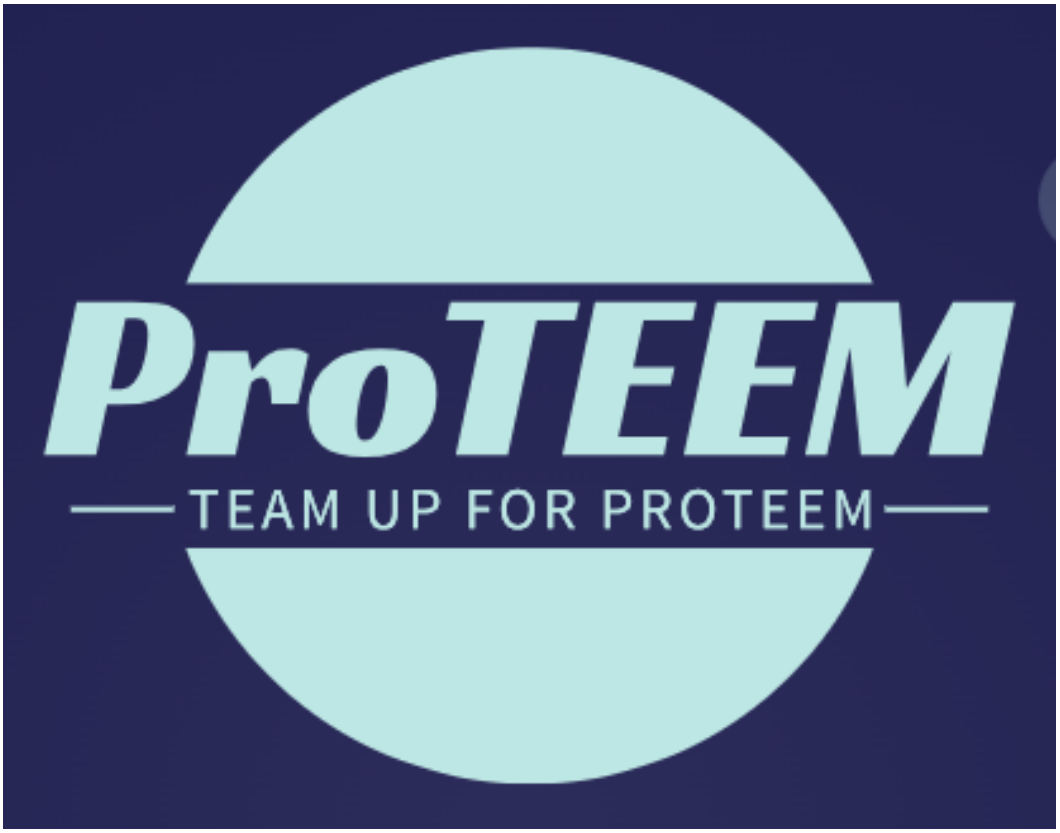

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CPD Module 2 Didactics of Mathematics

19-23 August 2024

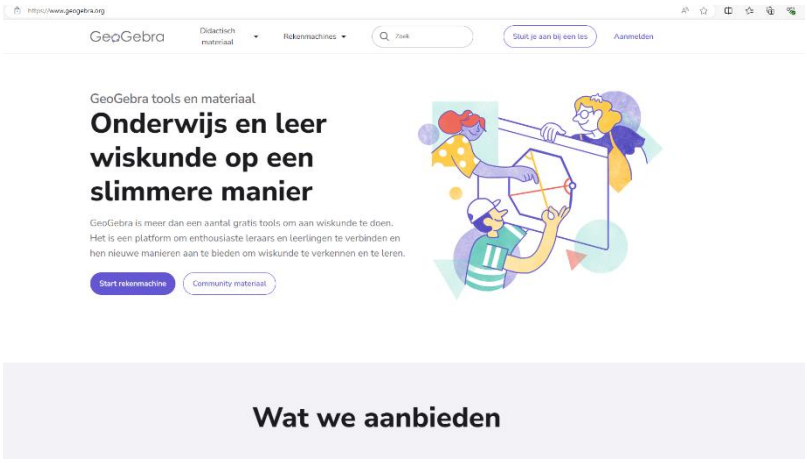
This course is created by G. Verbeeck, C. Maras and J. Busschots in August 2024 based on Chris Cambré's design in 2023. Gilberte Verbeeck made the English version.

Inhoudstafel

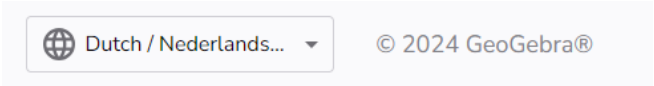
1. The GeoGebra Home page: geogebra.org

The GeoGebra Home page gives the overview of what you can do with GeoGebra. It's too much to list.

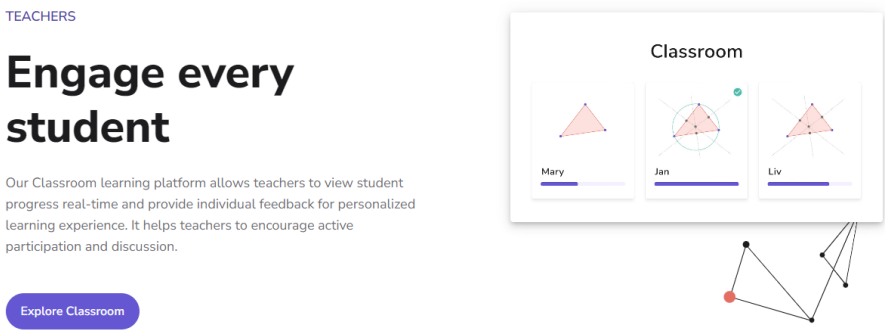
Go to **www. geogebra.org**. The screen below appears, if the language is set to 'Dutch'.



Do you want to use GeoGebra in another language? You can. Scroll to the very bottom of the page and set your language.



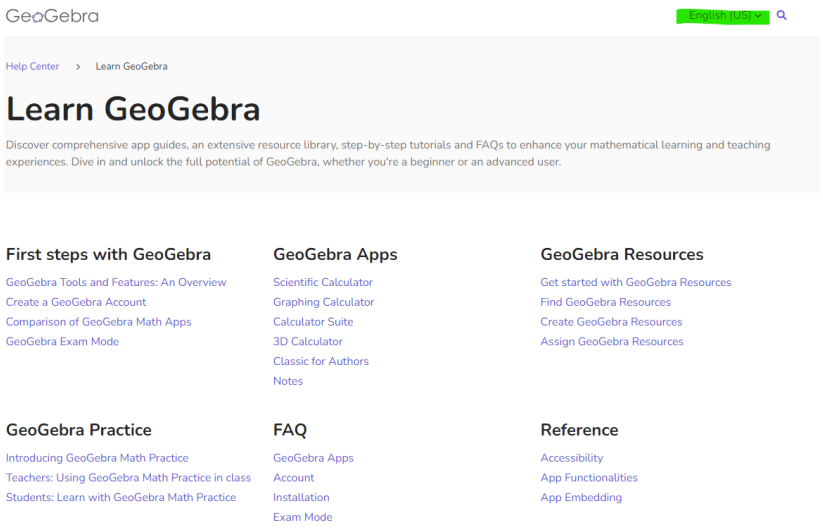
<u>Task:</u> On the home page, call up the following screenshots in English and French:



Scroll to 'Community materiaal' OR 'Math Resources' OR 'Ressources de la Communauté' (in French on the screenshot above). Click on 'Verken het allemaal' OR 'Explore all' OR 'Explorer tout'. Here you can find material that other teachers made and share with the community.

If you scroll down, you will find the ‘Help Centre’ at the bottom left.

If you then click ‘Leer GeoGebra’ OR ‘Learn GeoGebra’, it will take you to a page that takes you to all the initiation manuals. You can reset the language at the top.

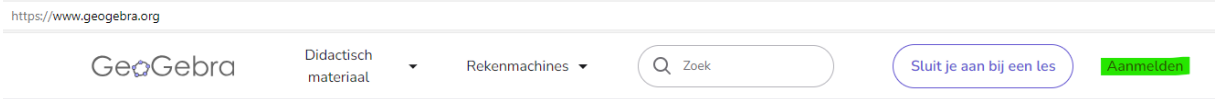


GeoGebra is very dynamic and constantly evolving therefore the screenshots in this course may be out of date from 2025 onwards. They were all taken in August 2024. Nearly all of them are in Dutch. We have enriched the screenshots with indications that support the text. In the different languages, the location of text and figures in GeoGebra is the same. We hope that this means readers will not find the Dutch language a problem.

2. De GeoGebra community.

Join the GeoGebra community by signing up and creating an account. After all, with an account you can create online materials, upload and organise your own GeoGebra files in GeoGebra books, assign materials to your students and share or publish your materials.

Assignment: Go to www.geogebra.org. Sign in using the button at the top right of the GeoGebra Website screen.



Als je aangemeld bent, verandert de bovenstaande balk op de openingspagina van GeoGebra:

Once logged in, the bar above on the home page of GeoGebra will change to:

You can read more about creating a GeoGebra account via this link. At the top right, you can change the language English (US)

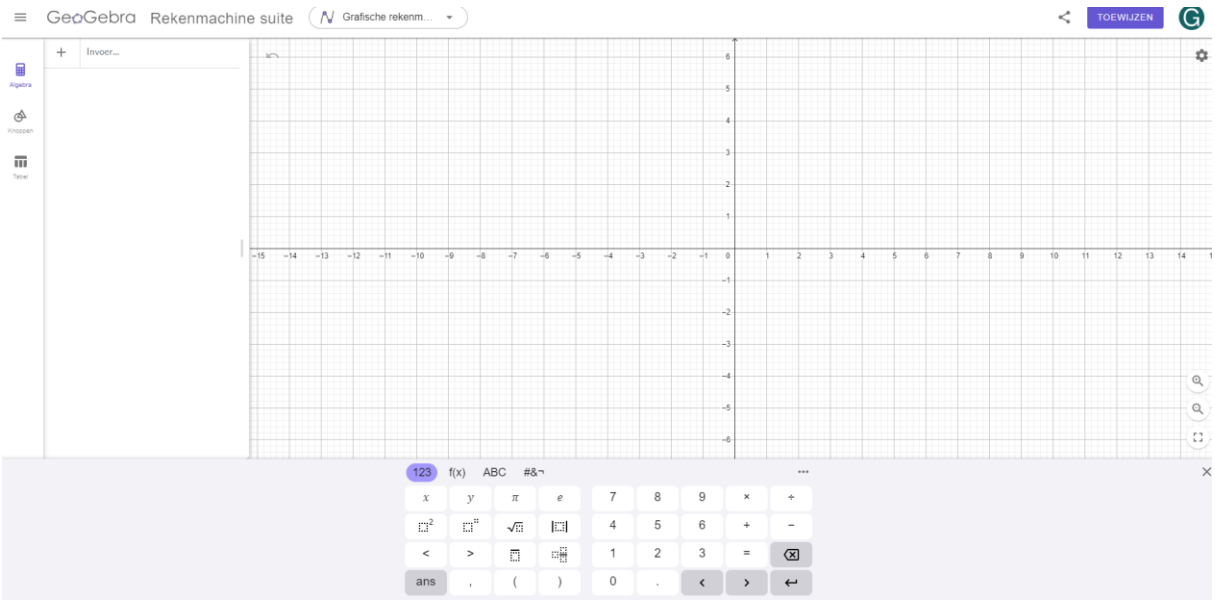
3. Calculator Suite

Calculator Suite is a bundle of several apps. You will find Suite under calculators. Suite is a powerful and intuitive graphical calculator.

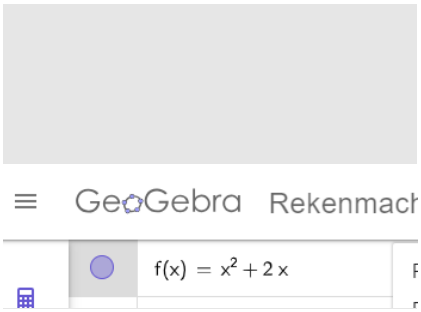
Click on 'Calculators' and 'Calculator suite'.

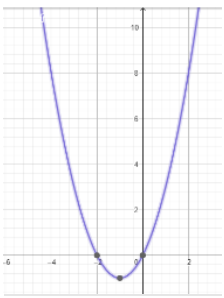
The Suite screen that opens is minimalist: a title bar, a vertical bar with a few icons **(Algebra, Tools, Table**) and one drawing window. You can make the drawing window bigger and smaller by moving the vertical line next to the icons.

Click the ***Algebra*** icon and an input field and (an icon of) a virtual keyboard appear.

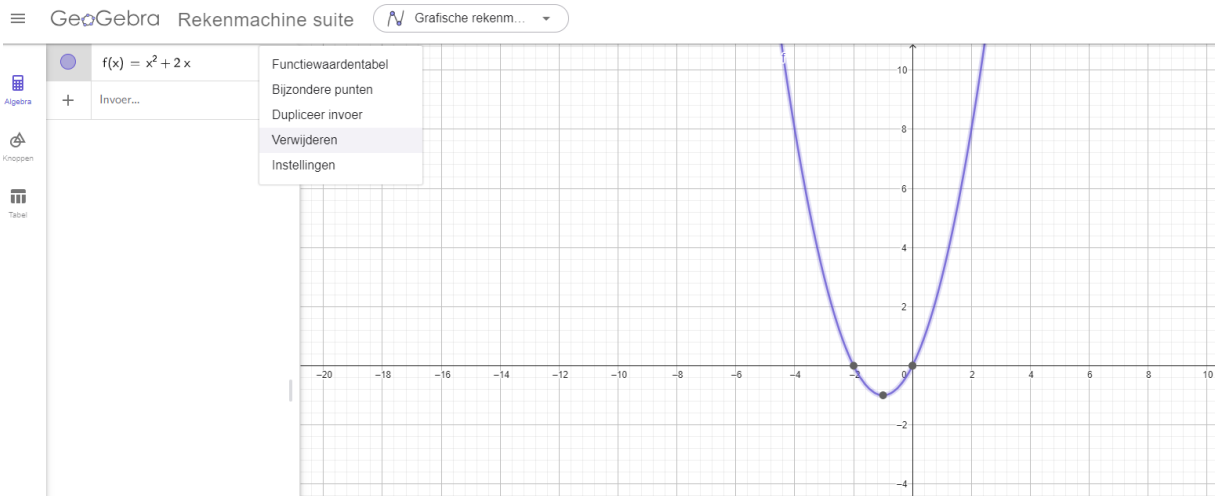


In the input field, you can type an algebraic expression e.g. $x^2 + 2x$. GeoGebra recognises this input as a function. In the drawing window, the associated function appears and in the input field the function gets a name.





Task: Try it out very briefly. Type an expression in the input field. You notice that three dots appear next to the function. Clicking on these shows what else GeoGebra can do with the function.



 With the virtual keyboard, you can smoothly type very diverse algebraic entries with root forms, fractions, built-in functions or mathematical operators. This keyboard becomes an icon when you click the little cross on the right.

Tip: If you want to work with fractions, type the edit character '/' in the input field. GeoGebra creates a fraction where you can enter the numerator and denominator (so you don't have to type parentheses).

Clicking the ***Algebra*** icon again closes the algebra window, the graph remains unless you delete the input field.

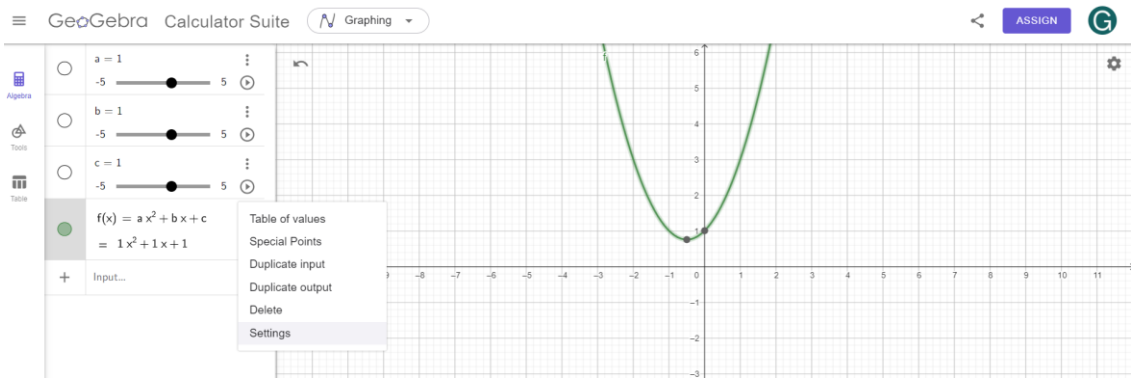
Click the ***Tools*** icon and a selection of buttons will appear, arranged by category: basic tools, edit, media, measure.... and if you scroll down, you will find *MORE*. Clicking on this will reveal all available tools.

Click on the ***Table*** icon and a function value table will appear.

3.1. An intelligent inputfield: sliders and operations.

Type $\boldsymbol{ax^2 + bx + c}$ in the input field. As you noticed above, GeoGebra turns this algebraic expression into an expression. Moreover, the graph appears in the drawing window. GeoGebra also automatically creates sliders for the parameters a, b and c. In the screenshot below, you can vary all three parameters between -5 and 5. By clicking on the little arrow next to the line segment, you make the parameter 'walk' and see the effect on the graph.

Clicking on the three dots shows even ***MORE*** additional possibilities. The screenshot below shows the options for the entered function.

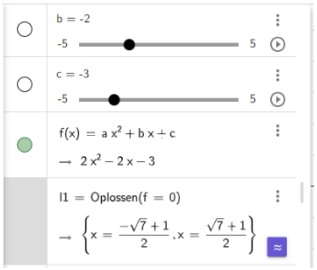


The option ***Special Points*** calculates the points of intersection with the axes and the extrema of the function. The option ***Table of values*** creates a table in the ***Table*** window. You notice that the ***Table*** window then opens.

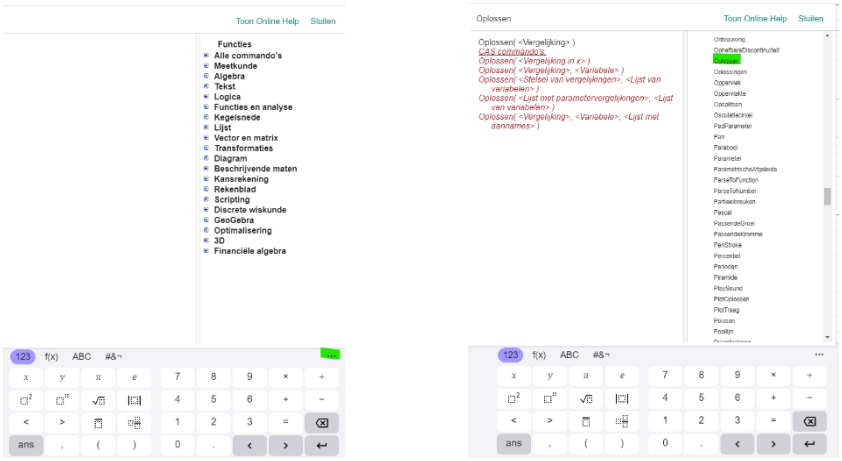
3.2. Operations via commands and built-in functions

Oplossen(f = 0).

In the input field you can enter commands to perform additional operations. For example, you calculate the exact value of any zero values with the command ***Oplossen(f = 0)*** (***Solve(f = 0)***).



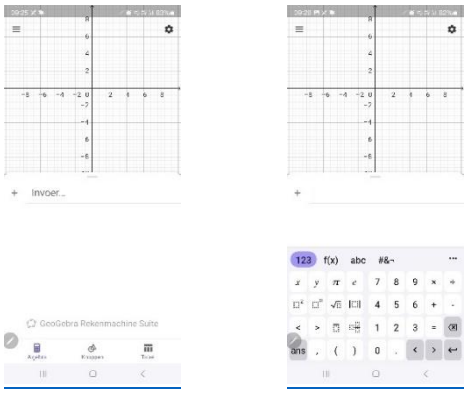
If you click on the three dots at the top right of the keyboard, a context menu opens with a list of all built-in functions and commands, arranged by category. If you select a command, the appropriate syntax appears.



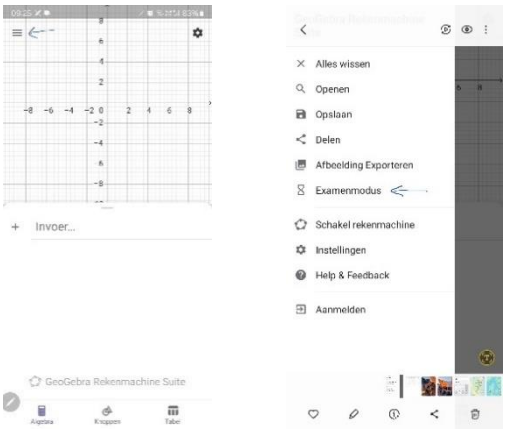
Clicking ***Toon Online Help*** (***Show Online Help***) links you to a wiki page with additional explanations with examples of a command and its possible syntaxes.

3.3. GeoGebra Calculator Suite on the smartphone

The app on the smartphone looks largely the same with the same features. As a result, Suite also presents itself as a graphical calculator for students and teachers.



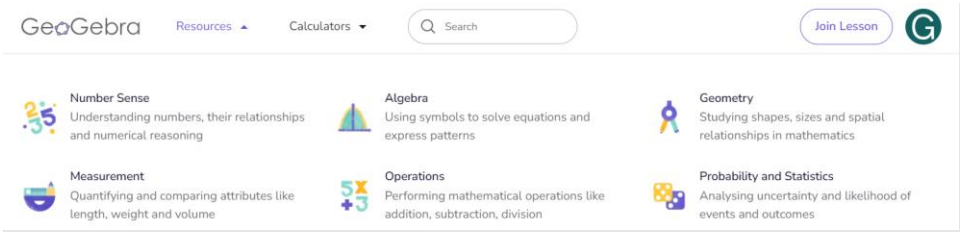
The app has an additional option for the use with tests and exams. Using the menu button at the top left of the screen (the 3 horizontal lines), you can access exam mode. This is a secure full-screen mode in which you can use the app as a graphing calculator during written tests and exams.



Full details on usage and security can be found via the opening page ***geogebra.org***. Explore 'Maths calculators & apps', scroll down to 'GeoGebra on tests' and click on 'Learn more about exam Modus'. You can also follow this link taking you to a Dutch page. At the top right, you can set the language.

4. Resources

GeoGebra offers resources. These are arranged by mathematical topic.

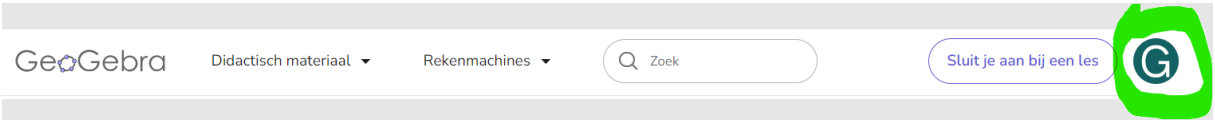


Many sample applets and worksheets are very thoughtfully designed. You can search for specific material via the 'Search' function. By typing e.g. 'Flink', you will find material from the Austrian FLINK project. Worksheets or applets in another language can be interesting. Adapting found didactic material is an interesting skill to learn. Besides creating a worksheet, adapting an existing one is covered further in this document.

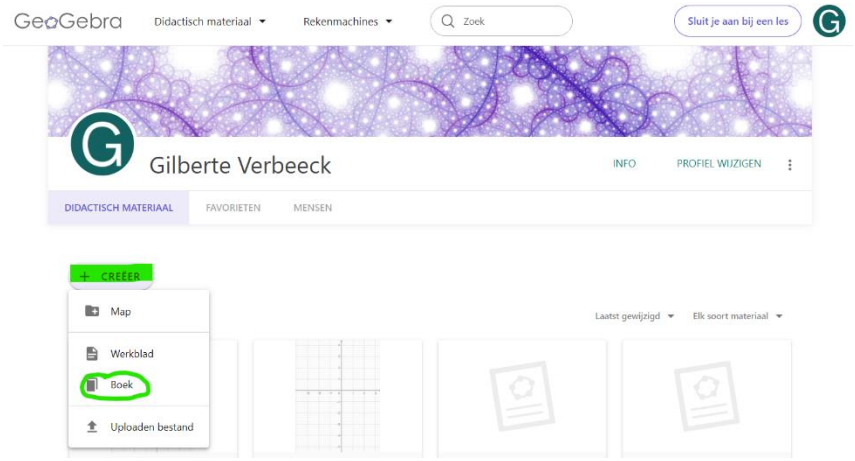
5. GeoGebra book

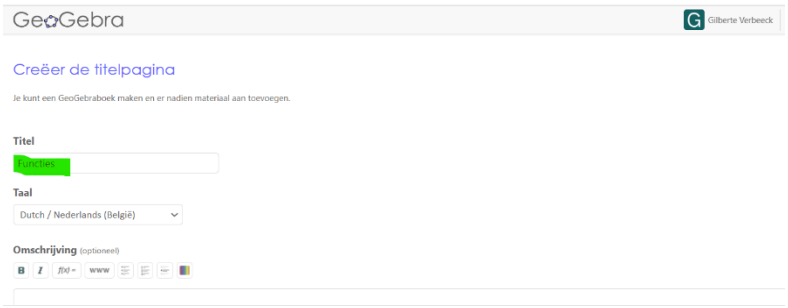
5.1. Creating a GeoGebra book

Task: Go to your profile page by clicking on the icon indicating your name.

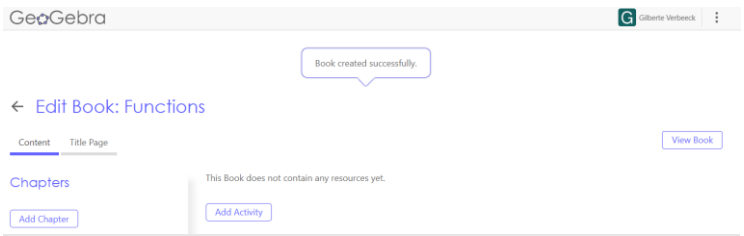


Click on the CREATE button and create a book titled ‘Functions’.



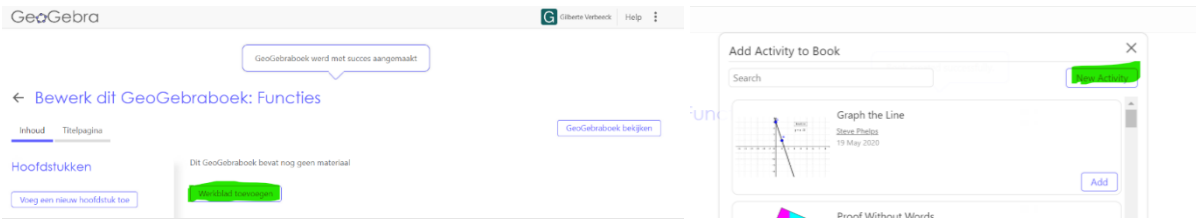


Scroll down and ignore Description, Target Group, Tags and Visibility. Click ‘Save’.

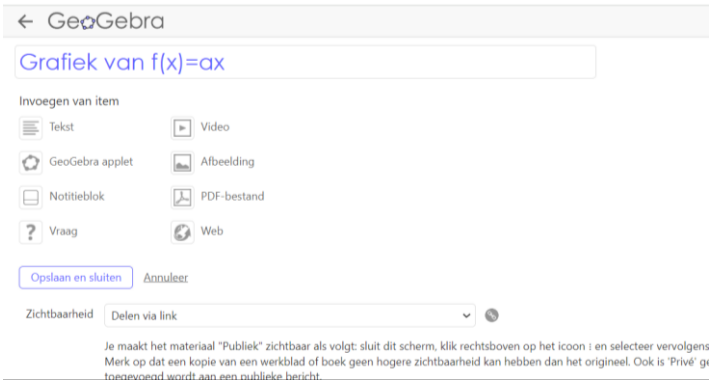


5.2. Create an activity (eigen werkblad) and add it to your Geogebra book

Add an activity (werkblad) to your book by clicking ‘Add activity’ and then ‘New activity’.

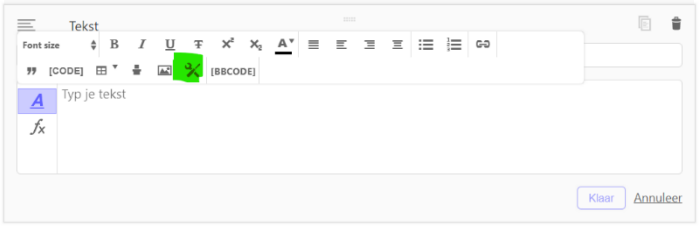


Give it the title 'Grafh of $f(x) = ax$'.



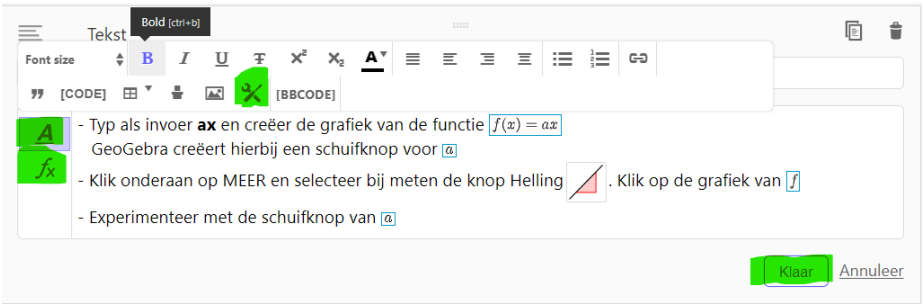
Insert a text box: Create a text box by clicking the 'Text' button (see screenshot above 'Tekst'). The screen in the screenshot below opens.

You need the buttons highlighted in green to bold a text, use the maths editor or choose a GG-icon.



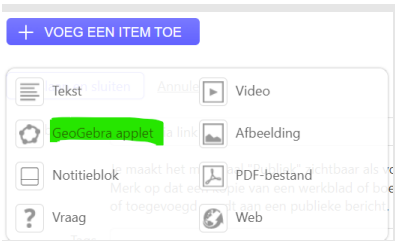
Enter the following text in the text box. You can copy this text as much as possible from this document.

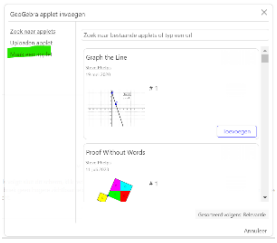
- Type **ax** as input and create the graph of the function $f(x) = ax$.
  GeoGebra hereby creates a slider for a
- Click MORE at the bottom and select the Slope button under Measure  . Click on the graph of f
- Experiment with the slider of a .



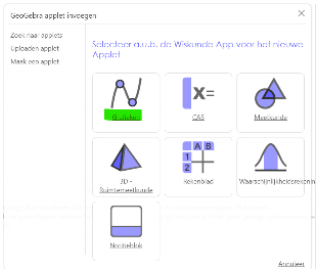
Click 'Done' ('Klaar').

<u>Insert applet</u>: Click ‘add element’ (‘voeg een item toe’) at the bottom and click ‘GeoGebra’ (‘GeoGebra applet’). You can choose from existing applets created by others. These appear on the right-hand side of the window. However, you are going to create your own applet, click ‘Create applet’.

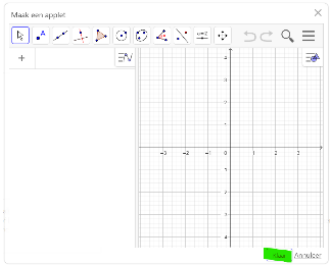


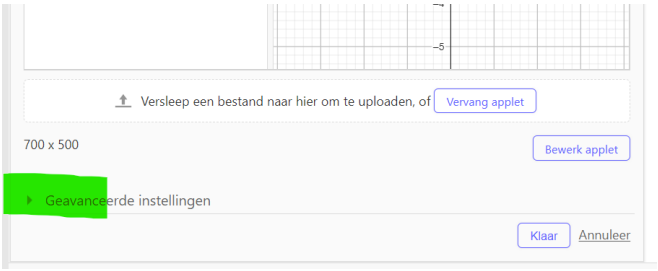


You have a choice of options. Click on ‘Graphs’ (‘Grafieken’).

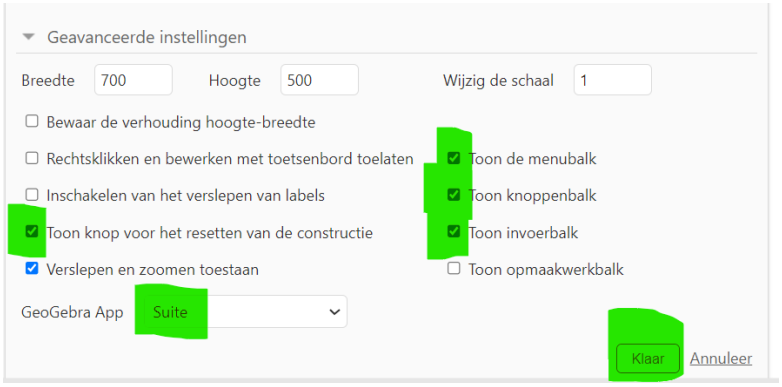


Click ‘Done’ (‘Klaar’) and then on the triangle below the applet so you can adjust the advanced settings (Geavanceerde instellingen).

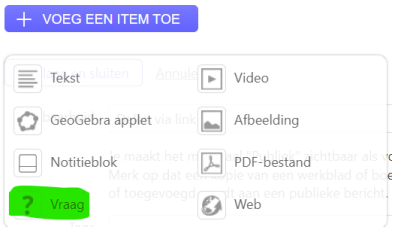


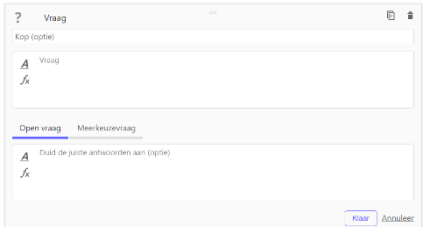


We adjust these settings so that students can create their own objects in this applet during class. To do this, choose ‘Suite’ and tick the boxes according to the screenshot below. Click ‘Done’ (‘Klaar’).



<u>Add multiple-choice question</u>: Click ‘Add element’ at the bottom. Click on ‘Question’ (‘Vraag’).

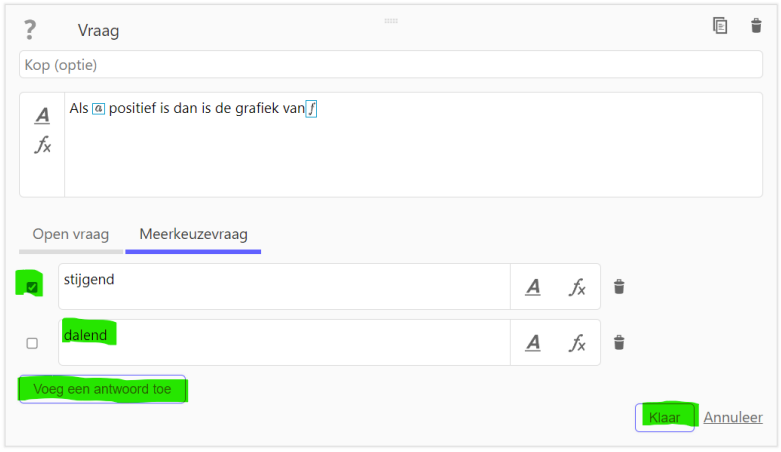




Complete the following: If a is positive then the graph of f is (Als a positief is dan is de grafiek van f)

Choose ‘Multiple choice’ (‘meerkeuzevraag’). Fill in the following first option: ‘increasing’ (‘stijgend’).

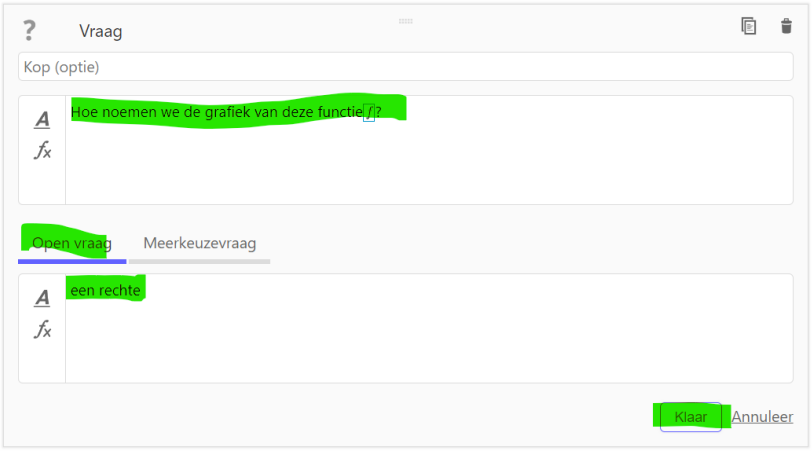
Click ‘Add Answer’ (‘Voeg een antwoord toe’) and enter: ‘decreasing’ (‘dalend’). Tick the correct answer. Finish again by clicking ‘Done’.



Add open question: Click on ‘Add element’ at the bottom. Click on ‘Question’.

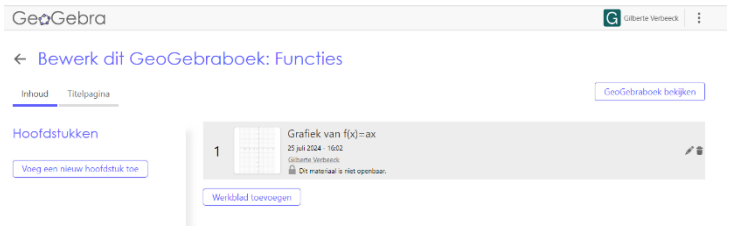
Enter the following: ‘What do we call the graph of this function f ?’ (‘Hoe noemen we de grafiek van deze functie f ?

Choose ‘Open question’ (‘open vraag’). Fill in the answer: ‘a line’ (‘een rechte’). Finish again by clicking ‘done’.



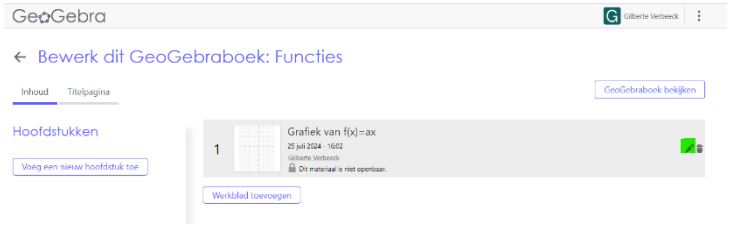
Save Activity: Click ‘Save and Close’ (‘Opslaan en sluiten’).

Test the activity (‘werkblad’): by clicking on it

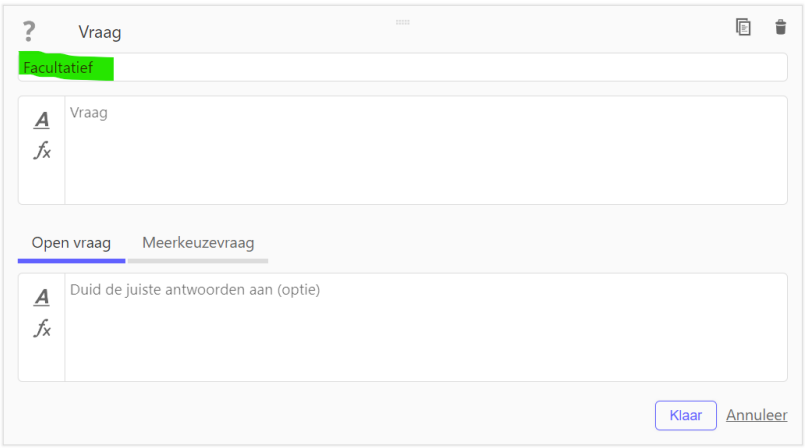


Optional if you have time to spare: add an open question with multiple correct answers.

Edit the activity you just saved by clicking on the pen.



Click on ‘Add element’ at the bottom. Click on ‘Question’. Enter ‘Optional’ (‘Facultatief’) in the header. This indicates that this question is optional for students. This is a form of differentiation. Fast learners can learn or practise something extra that is not strictly necessary according to the curriculum.

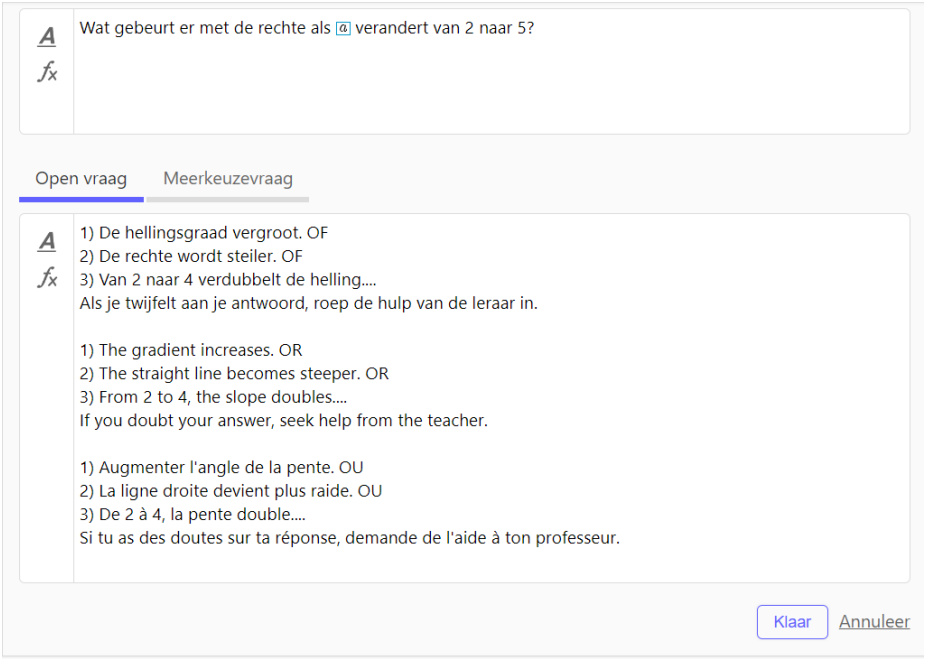


Complete the following question: ‘What happens to the line if a changes from 2 to 5?’ (‘Wat gebeurt er met de rechte als a verandert van 2 naar 5?’)

Choose ‘Open question’. Fill in the possible answers:

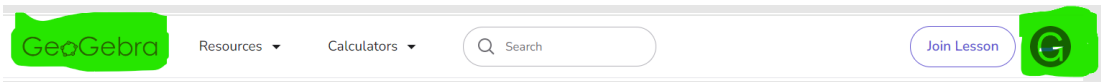
1) The slope increases. OR
2) The line becomes steeper. OR
3) From 2 to 4, the slope doubles....

If you doubt your answer, seek help from the teacher. Vul de volgende vraag in:

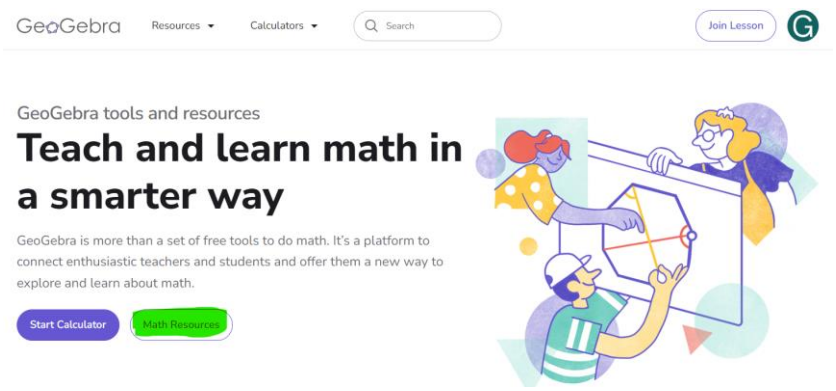


5.3. Adding an existing worksheet to your Geogebra book

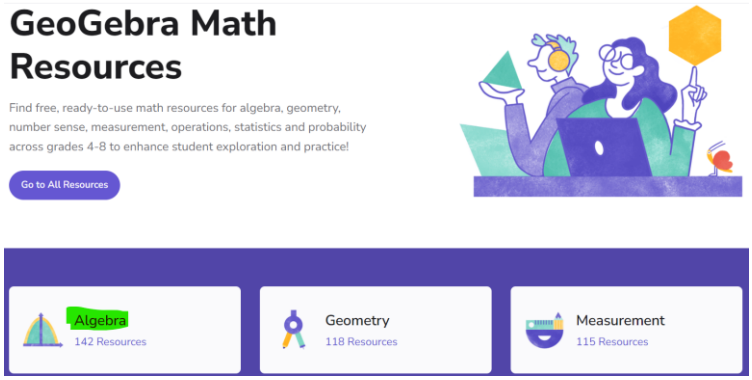
Go to the opening page of GeoGebra. From your profile page, do so by clicking on the icon referring to your name and then on GeoGebra.



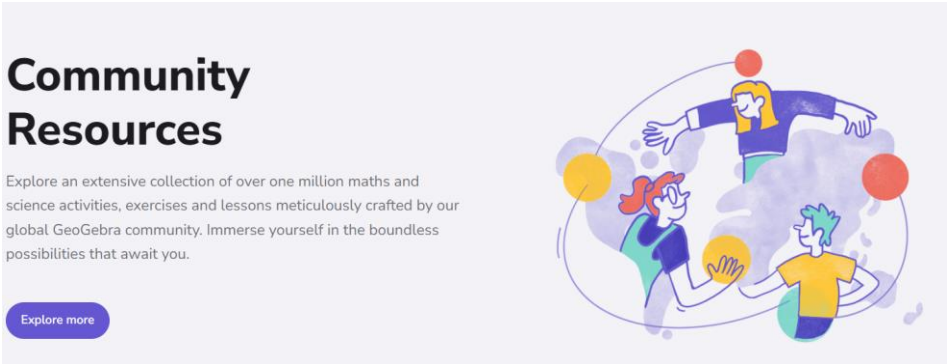
Click on '**Math Resources'**.



The screenshot below shows the top of the page. Clicking here will take you to applets intended mainly for primary and lower secondary schools. If you click on Algebra, for example, you can choose from 142 applets arranged by subject and with an indication of the grade for which it is suitable.

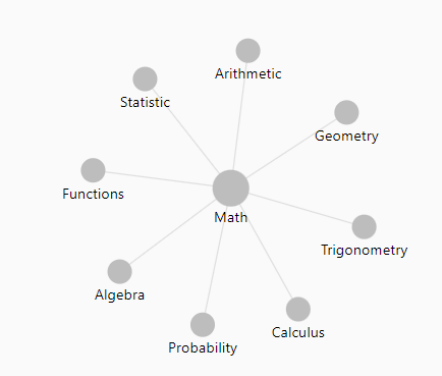


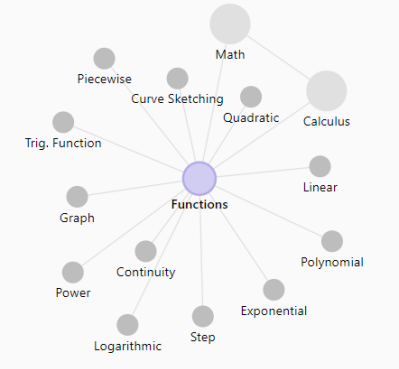
If you scroll down you will find the screenshot below.



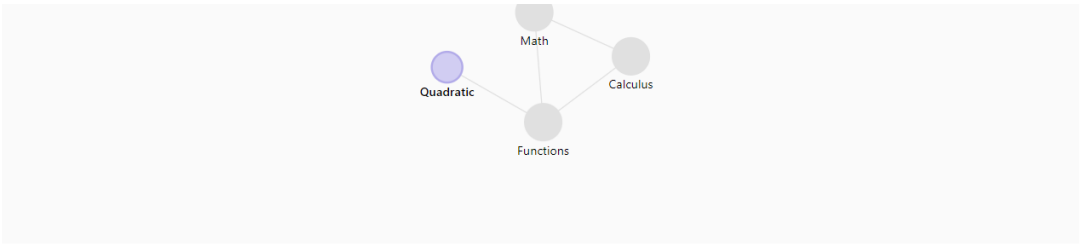
Click on 'Explore more'.

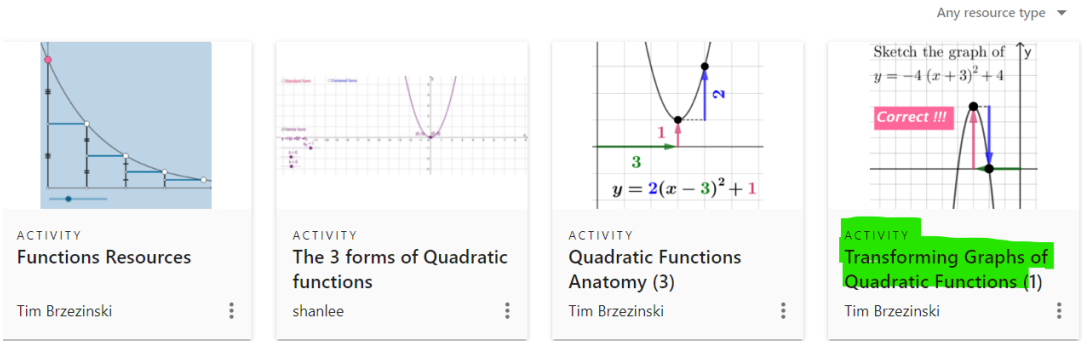
A lot of topics appear. By clicking further, you can refine the topic and find a suitable applet. In the example below, we choose '**Functions**' and then '**Quadratic**'.





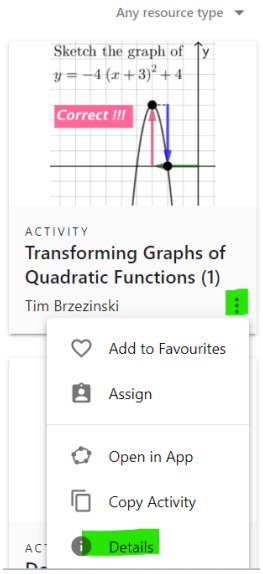
Check how the applet '**Transforming graphs of a quadratic function (1)**' works



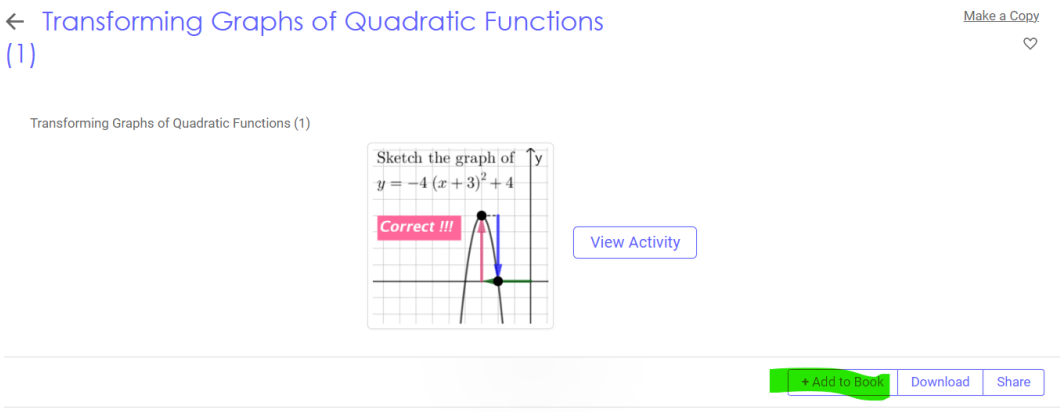


We are going to add this worksheet to the GeoGebra book.

To do so, click on the 3 dots (MORE) and select the **Details** option.



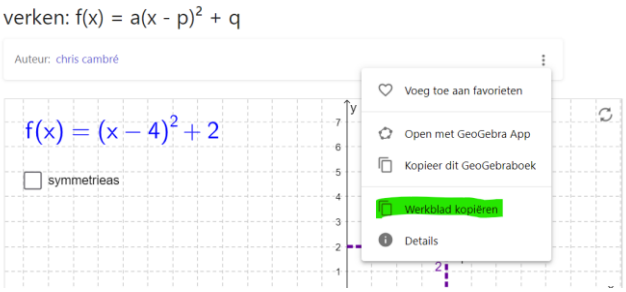
The following screen appears. Press the button ‘Add to Book’.



You now enter a dialogue box in which the books you created appear. Choose the book Functions, to which you want to add this file.

5.4. Modifying an existing activity

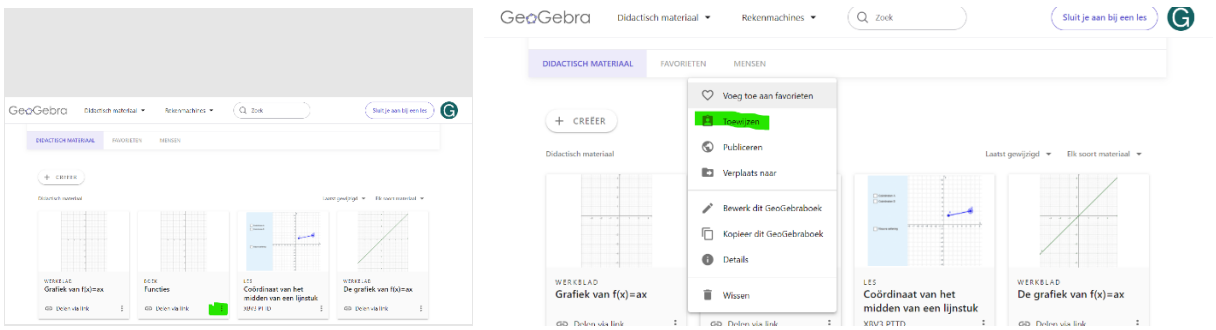
You can also choose to add a modified version of an existing activity to your book. Then select the Copy Activity (Werkblad kopiëren) option.



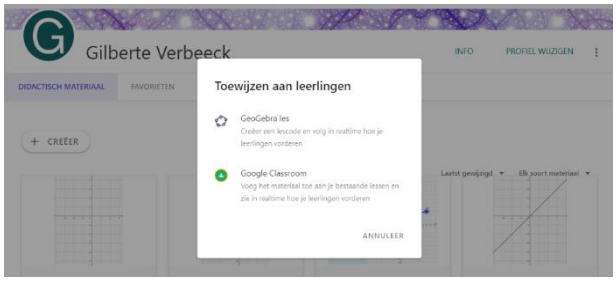
Make the preferred changes and save the modified version in your own account. Then you can add this customised version to your book.

5.5. Create a lesson

Click the 3 vertical dots next to the GeoGebra book you want to use in your lesson. Click the ***Assign*** (***Toewijzen***) button.

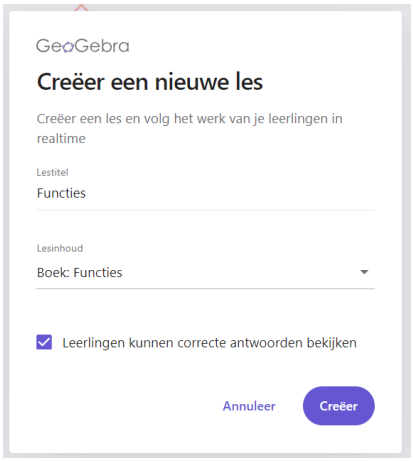


You can choose between a GeoGebra lesson or Google Classroom.



Google Classroom is an application from Google in which you can create class groups for the classes you teach. Within this application, you can monitor student progress. It is a tool within which you can integrate GeoGebra. We won't go into detail. You can find more explanation at: https://sites.google.com/view/classroom-workspace/

Clicking on 'GeoGebra lesson' produces the following screen



Clicking Create (bottom right) automatically brings you into the Classroom (Klaslokaal). GeoGebra generates a code that gives your students access to the GeoGebra book. They can now use it.

How you can work with this in your lesson you will read in the next section.

6. GeoGebra Classroom

6.1. How a lesson proceeds

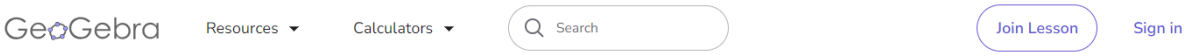
6.1.1. Learner joins a lesson

Method 1: Learners enter a given code to join.

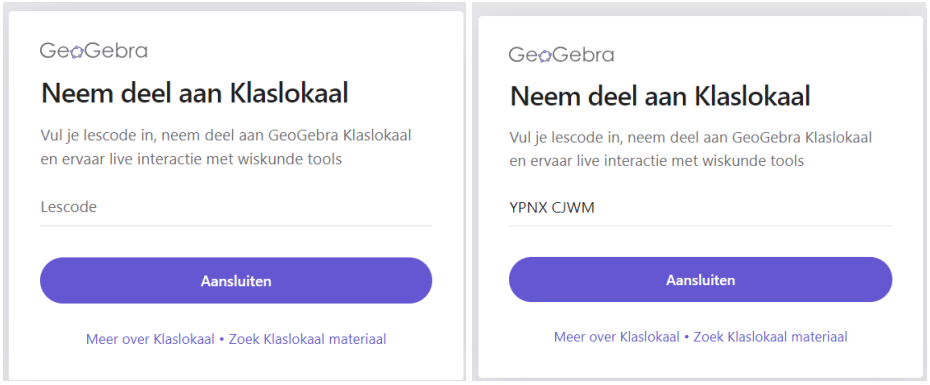
Display the opening page of the lesson with the code to the lesson.

Give the following instruction to your learners

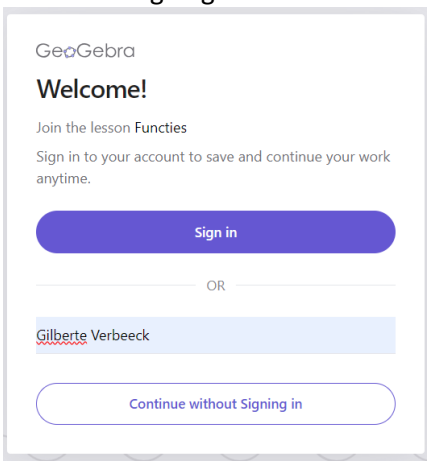
- Go to www.geogebra.org and click on 'Join Lesson' ('Sluit aan bij een les').



- In the dialogue box, enter the ***lesson code*** (***lescode***) for your GeoGebra lesson page.



- Enter your ***name*** and continue without Signing in. You are now in class!



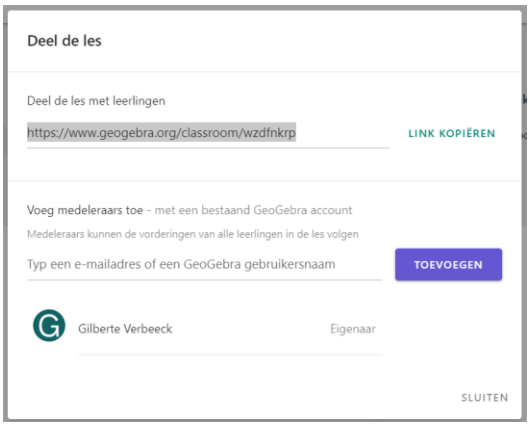
Learners can participate without a GeoGebra account. But they can only leave a lesson and continue working later if they joined the lesson from an account. Indeed, work can only be saved in their own account.

Method 2: The teacher provides the learners with the link

After creating a lesson, you got the following screen:

Clicking on the share icon with the code at the top right 

, will take you to the screen below. You can share the link with learners and add co-teachers via e-mail addresses.



You can provide via email, the online learning platform, whatsapp… each learner with the link to join the lesson. In this option, learners themselves do not have to enter a Lesson code but they do have to enter their name.

6.1.2. The learner follows the lesson.

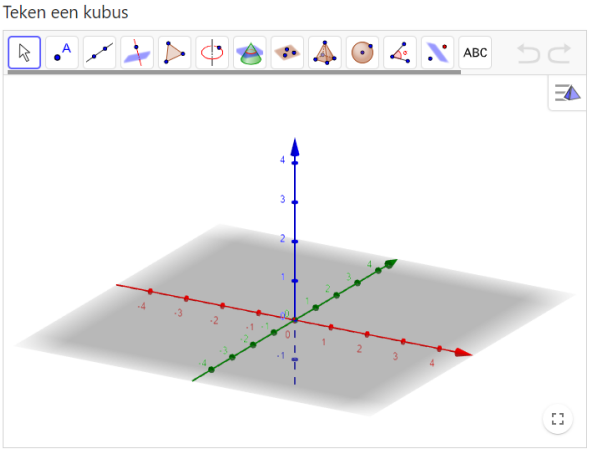
Give your students the following instruction:

You will find tasks in the lesson. This may be in the form of:

- a do-applet in which you have to make your own construction, deduce connections ...;
- multiple-choice questions in which you have to tick all the correct answers;
- open questions in which you have to formulate an answer yourself.

Do-Applet

In an applet, you can use commands or buttons to create a construction. This construction can then help answer additional questions in the lesson. Below is a screenshot in an applet where you are asked to draw a cube (teken een kubus).

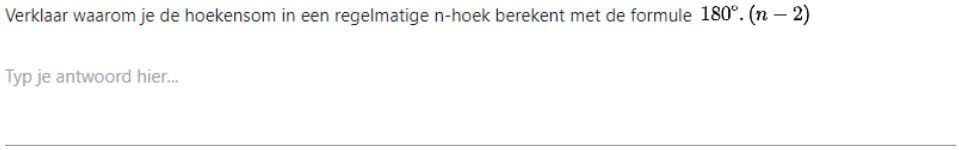


Multiple-choice question

For a multiple-choice question, tick the square with the correct answer. Below is a screenshot with a question about the number of faces of a cube (aantal zijvlakken van een kubus).

Open-ended question

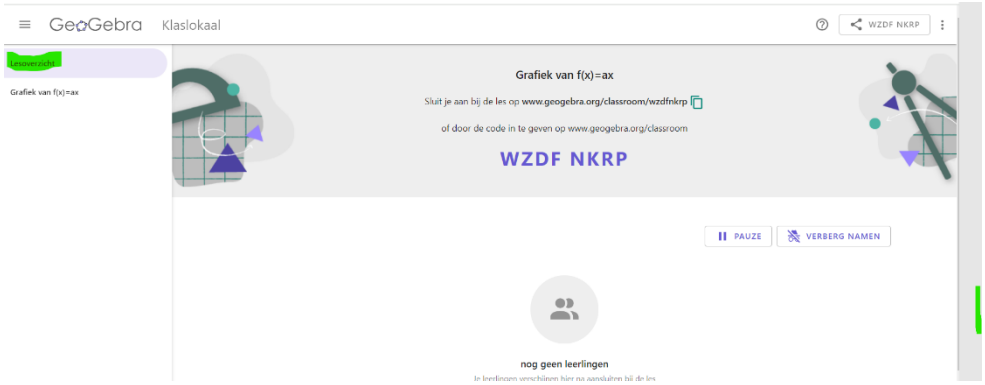
In an open question, you formulate the correct answer. An example here concerns the angle sum in a regular polygon (hoekensom in een regelmatige n-hoek). After constructing a triangle, quadrangle, pentagon... in an applet and calculating the angle sum each time, you are asked to find a general formula for the angle sum in a regular polygon. If you did not find it, you are given the next open-ended question: 'Explain why you calculate the angle sum in a regular n-polygon with the formula $180° \cdot (n-2)$



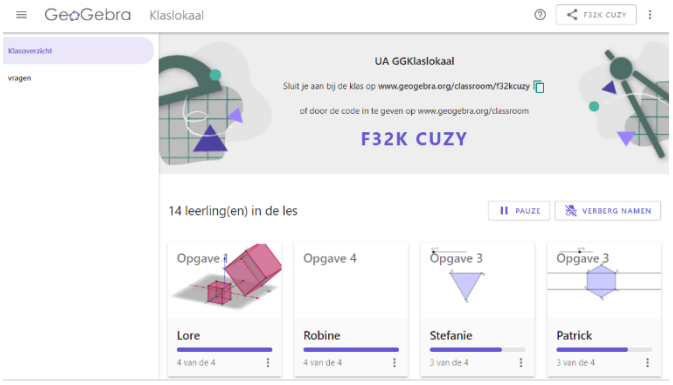
The prescribed answer to an open-ended question may differ from what you wrote down yourself. If you are in doubt whether your answer is correct, ask your teacher to assess it.

6.1.3. Progress of a lesson

At the top of the lesson's teacher screen you will find the *Lesson Overview* page (*Klasoverzicht* of *Lesoverzicht*).



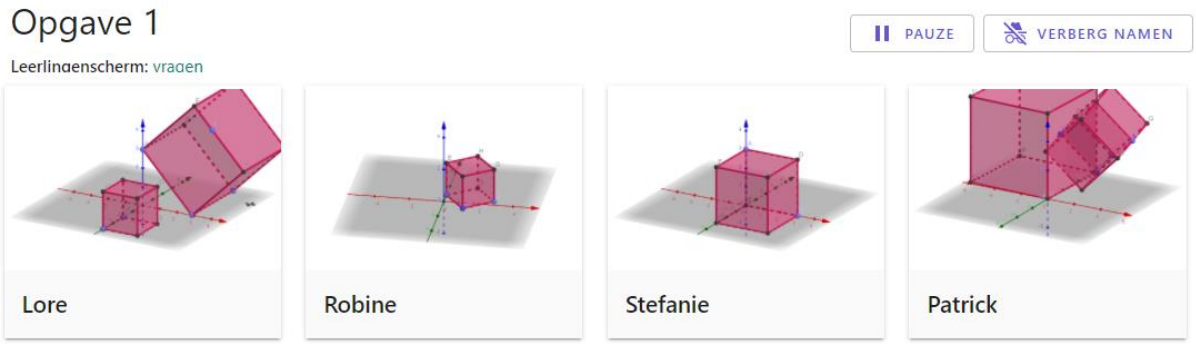
In the Lesson overview you can track how many students are participating in your lesson and how many tasks they have already worked on. The images update every 2 seconds. In the screen above, no students are participating yet. In the screen below, 14 learners are taking part.



In this overview, you will find an overview of the tasks for each activity. If you click on a task, you can follow the progress of all learners on the task in real time. Clicking on one of the thumbnails shows you the learners' applet in full size.

Progress in constructions

The screenshot below shows the constructions of students Lore, Robine, Stefanie and Patrick.



You can hide learners' names. To do so, click the ***HIDE NAMES*** button in the top right corner. This allows you to show all learners' work in a class discussion. Class climate is important for this. For example, a learner should not feel embarrassed for a wrong answer. By hiding names you avoid any problems of this nature. By selecting the ***DISPLAY NAMES*** button you show the learners' names again.

The pause button allows you to pause the lesson. Learners can only continue working on tasks when you click the ***REPEAT*** button.

Answers to questions

Go to the ***Overview page*** again. Select a multiple-choice question. When learners answer a question or change their answers, you immediately see the class results in a bar chart. Learner answers to open-ended questions also change immediately in real time and can be shown anonymously to anyone.

6.2. How to create a lesson

You can, even if you are not the author, turn any public online activity or book into a lesson. As briefly described in Creating a Lesson, you do this by clicking the **ASSIGN** button in the material you want to share. You can build the lesson clearly by creating a book where you spread the applets and questions over several activities. When following the lesson, students follow the lesson activity by activity, following the structure of the book. For the teacher, the tasks are numbered across the different activities. In the Lesson Overview, you can oversee tasks activity by acitivity.

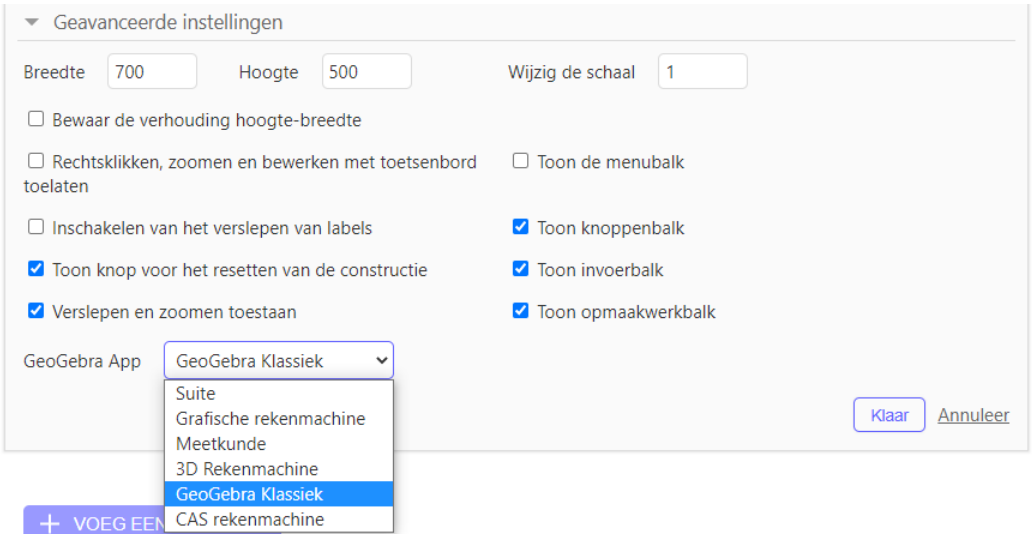
In Creating Lessons, we also mentioned ***Google Classroom***, a learning environment in which you can create classes in advance. If you check the option Google Classroom in the Assign button, you combine the advantages of both applications: Learners do not have to log in individually each time via the Lesson code and they can submit assignment work in Google Classroom.

We repeat and complement some things that are interesting if you are creating lessons

Applets

When you save a GeoGebra file in GG Suite, you create an activity. You'll find it on your GeoGebra profile page. You can edit an activity and add other elements to it (text, questions, etc). You can copy an activity from another author. Then you can make your own version of it. For example, you can also translate an activity into another language. An applet is only useful within Classroom if a learner can do more with it than just look at it. When creating or uploading an applet in an activity, you can define the applet's usage options and appearance in the ***Advanced Settings***.

- Move the mouse pointer over the applet and click the ***Edit*** pen icon.
- Under the applet, click the triangle and open the ***Advanced settings***.
- In the left-hand column you determine the user options in the drawing window.
- In the right-hand column, you determine which extras are shown next to the drawing window. If you show the ***input bar*** and ***button bar***, a user can use commands and buttons to create additional objects in the applet.
- At the bottom, you decide whether the applet appears in GeoGebra Classic or Suite or in one of the apps. This way, you can let learners work in the GeoGebra interface they are used to.

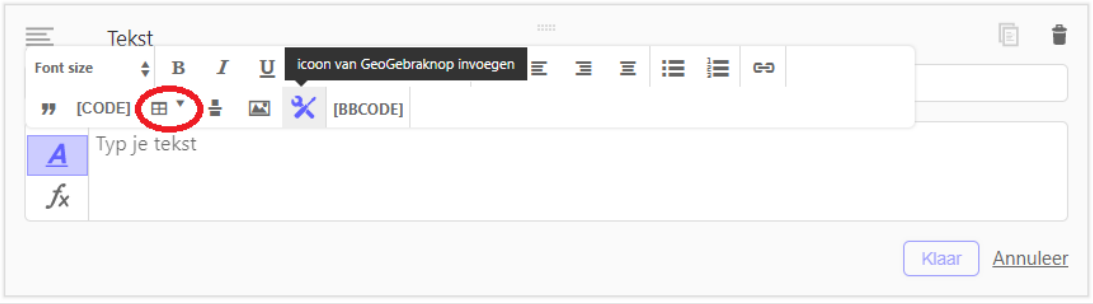


In a lesson, you can have learners start in a blank window, instruct them to manipulate existing objects or create additional objects in an applet with the initial situation prepared.

Step-by-step plan

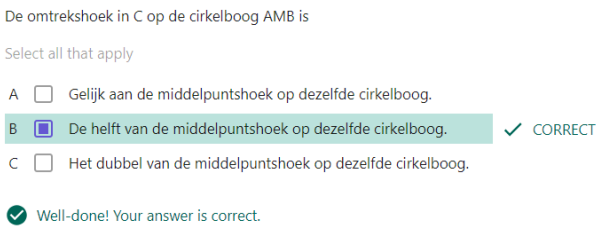
In a text box, you can have a learner follow a step-by-step plan. Some interesting buttons:

- Click the letter symbol A to open the formatting window.
- Click the buttons icon to insert a GeoGebra button icon in the text.
- In a table, you can nicely align step numbers, button icons and descriptions.



Questions

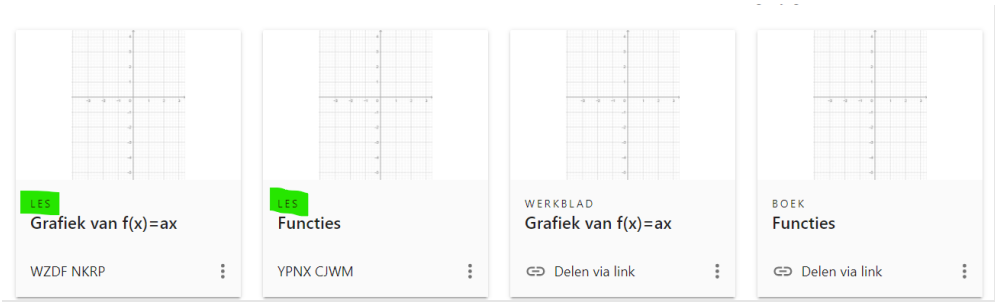
You can create multiple-choice and open-ended questions. The learner gets three answer probabilities. His answer to a multiple-choice question is immediately evaluated.



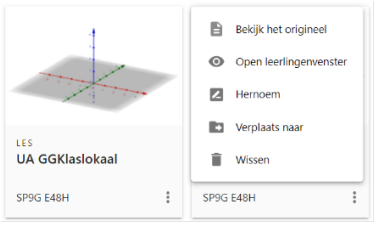
For open questions, a formula editor is available in the answer field. The answer to an open question is not evaluated. However, as a teacher you can formulate a model answer that a student can compare with his own answer.

6.3. Organising and editing lessons

On your profile page, lessons form a separate category for easy retrieval.

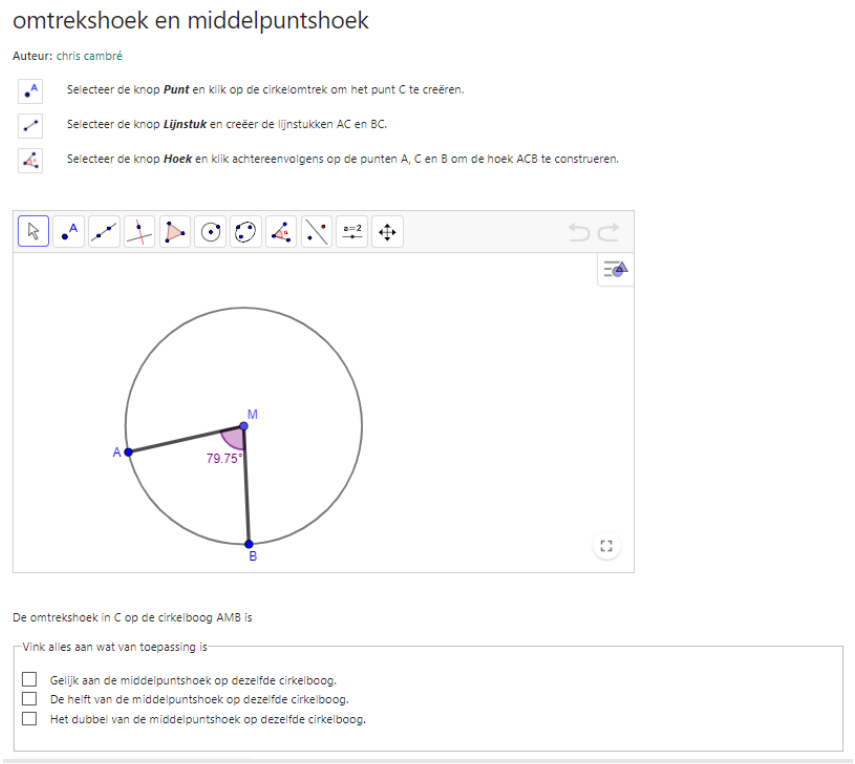


You can organise your created lessons into folders. To do this, click the 3 dots (More) by the lesson's thumbnail image and select ***Move to (Verplaats naar)***.



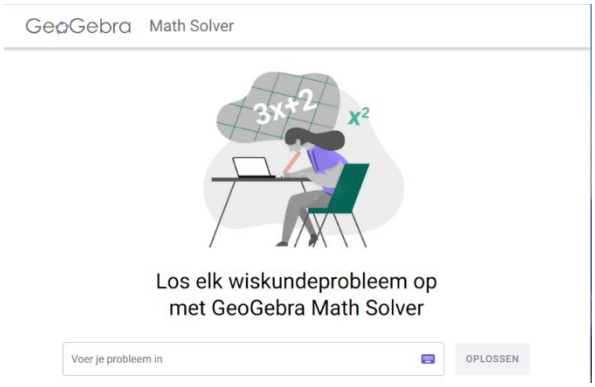
If you select ***View original***, the lesson's source file opens. You can create a new lesson from it for a new class group. You can also modify the source file. The changes also appear in the lesson(s) of the already created lessons. This allows you to fix errors, add a problem or a activity with the solution(s) of your lesson.

An example of an activity (werkblad) with a step-by-step plan, a start file and an accompanying question can be found at https://www.geogebra.org/m/t39b8ugj. In this activity, learners follow a step-by-step plan, investigate the relationship between a perimeter angle and a midpoint angle on the same arc and answer a multiple-choice question.



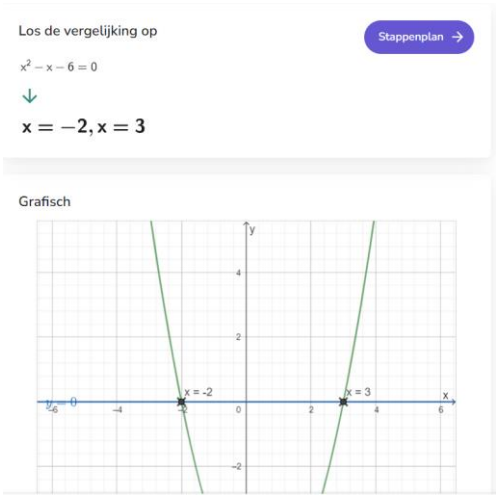
7. The new GeoGebra math solver

GeoGebra developed a new math solver. Like other solvers, it works out mathematical expressions and solves equations. But the interesting thing about this GG solver is that it shows multiple solution methods and you can zoom in on the intermediate steps in detail. Access Math Solver via [GeoGebra Wiskunde oplosser - stapsgewijze oplosser van problemen]. Again you can change the language on the bottom of the page.



7.1. Example 1 – quadratic equation

To solve the quadratic equation $x^2 - x - 6 = 0$, enter this equation in the 'Enter your problem' ('Voer je probleem in') field. Click solve and you will get the screen below.

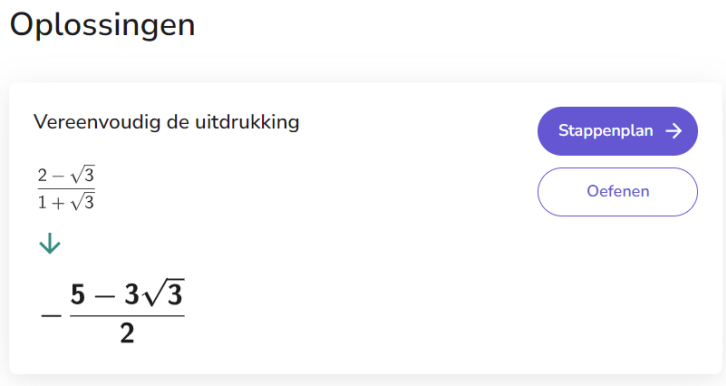


Clicking on 'solving steps' ('stappenplan') will give you the steps leading to the solution.

7.2. Example 2 - making the denominator root-free

The solver makes a denominator root-free of an entered expression and simplifies the form. In the input field, enter the fraction as shown in the screenshot below.

Click on 'Solve' ('oplossen') and the solution appears.



Clicking on 'Solving steps' ('Stappenplan') and 'Explain how' ('Leg uit hoe'), you can visualise different steps in the solution with explanations of the steps.