ProTEEM

TEAM UP FOR PROTEEM

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Module 2: Didactics in Mathematics – Day 1 Part I

August, 19-23, Antwerp

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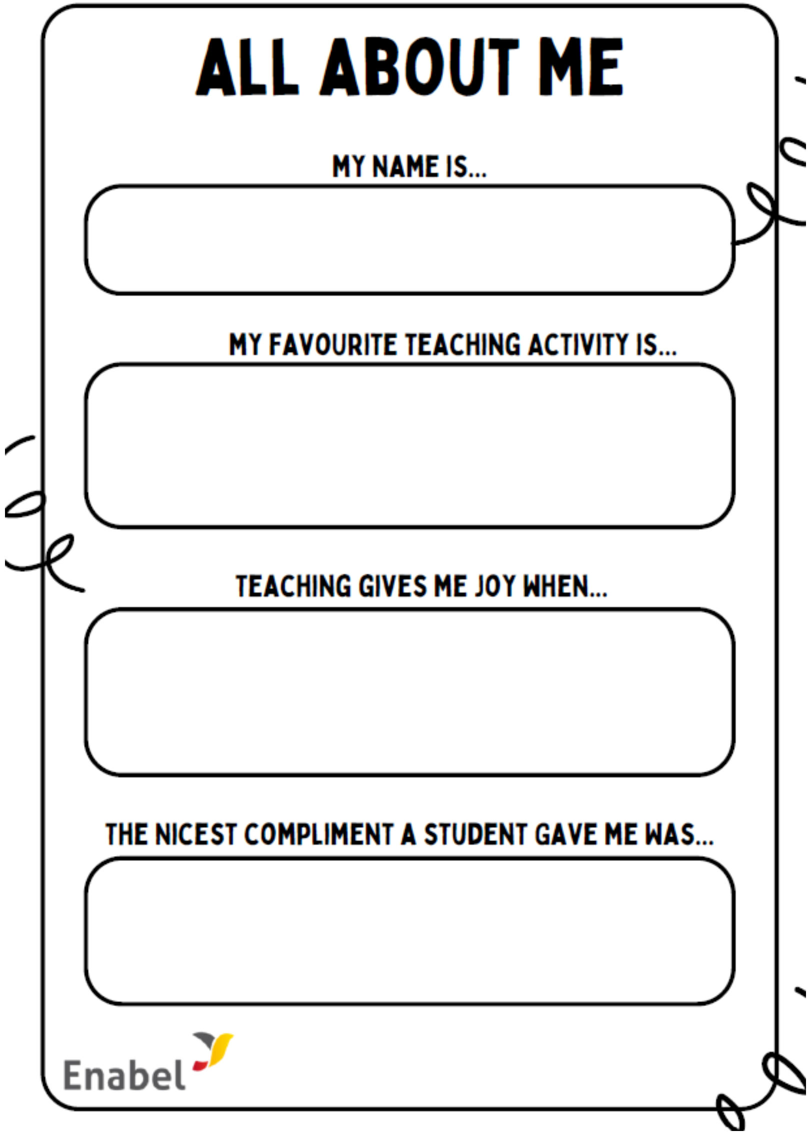
Ice breaker

- Briefly introduce yourself to the group

  All about me

- When will this week be a success for you?


ALL ABOUT ME

MY NAME IS...

MY FAVOURITE TEACHING ACTIVITY IS...

TEACHING GIVES ME JOY WHEN...

THE NICEST COMPLIMENT A STUDENT GAVE ME WAS...

Enabel

Good mathematics education

Brainstorm:

What does
"good mathematics education"
mean to you?

- Go to the padlet: https://padlet.com/elstanghe/good-mathematics-education-89r1bgydwqxcrhxa
- Click on the coloured circle in the bottom right-hand corner of the screen and add your input

Case

- During the lessons about second-degree functions, Mrs. Peeters ensures that there are many exercises in which the students have to make a sign chart, e.g.
    - Make a sign chart to analyze the functions behavior if $f(x) = 2x^2 + 2x - 4$
    - Method?
    - Calculate zeros via discriminant, enter them in the sign table and enter characters

- Her approach seems to work, because the students can soon easily draw up sign charts of 2nd degree functions

Case

- As **a homework task**, Mrs. Peeters asks to do the following exercise:

  You are throwing a ball with a friend. You throw the ball according to the function $y = -x^2 + 5x$*. It ends up neatly in your friend's arms. How far apart are you?*

- Almost **no one** in the class manages to complete the exercise. How come?

Think – Pair - Share

Case

- Based on the exercises in class, students have **mainly learned to apply a fixed procedure/technique** (cf. conducting drawing research).

- Students need more than that in their homework:
  - Translate problem into mathematical formula
  - Check whether it is a second-degree function
  - See which solution strategy can be chosen
  - Feel like getting stuck into the problem
  - ...

Mathematical proficiency

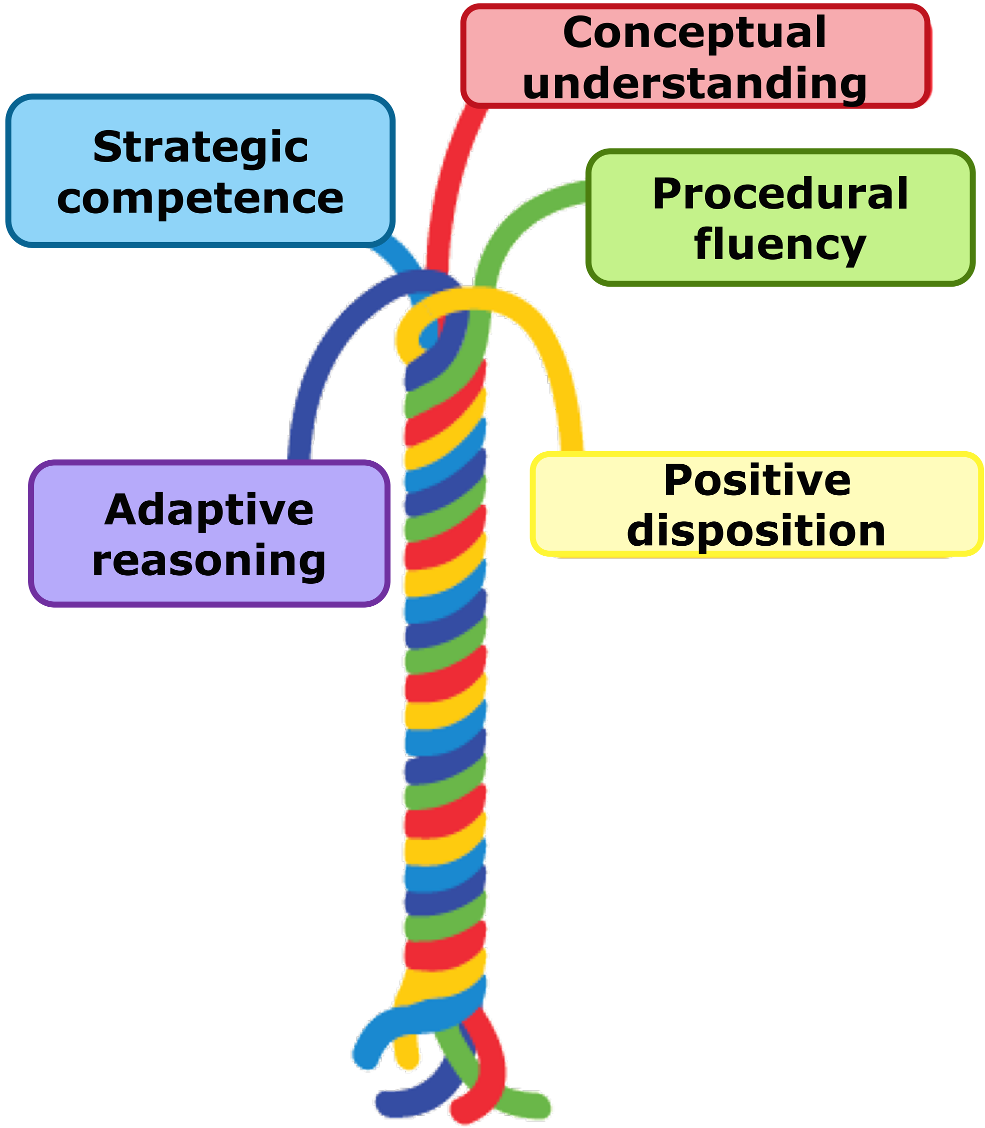
- You need more than **procedural knowledge** to be able to handle mathematics smoothly (as a student and as a teacher)!

- Kilpatrick and colleagues mapped this out:
  - Kilpatrick, Jeremy, Swaord, Jane, & Findell, Bradford (2001). Adding It Up: Helping Children Learn Mathematics. Washington, DC: The National Academies Press.

- **Kilpatrick's model: mathematical proficiency consists of 5 'strands' that are intertwined with each other**

- **Only someone who is familiar with each of these strands can handle mathematics fluently**

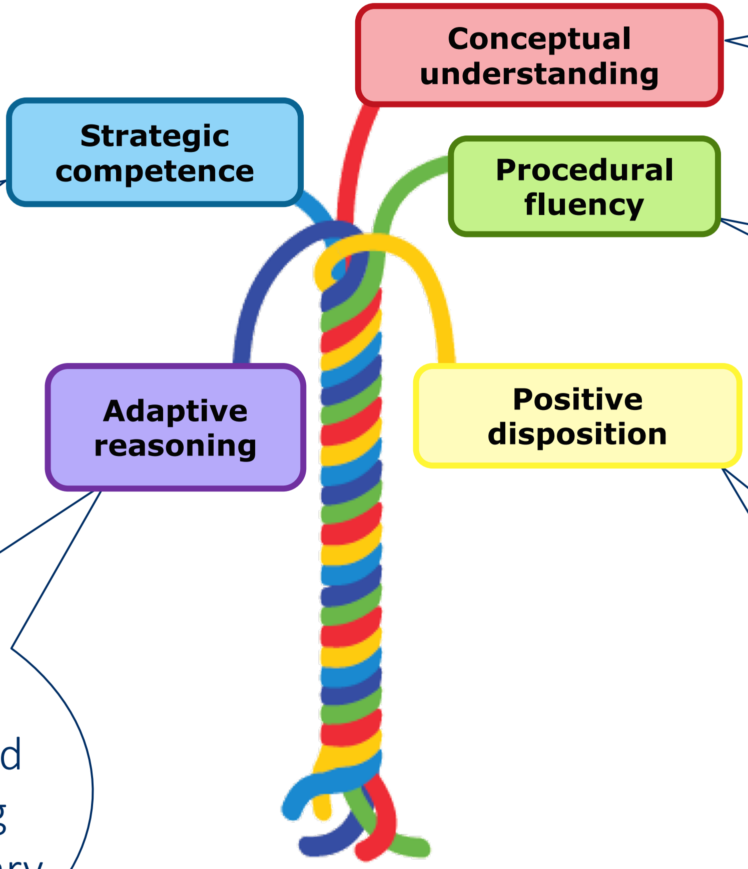
Kilpatrick's model



- **Conceptual understanding:** understanding mathematical concepts, operations and relationships
- **Procedural fluency:** knowledge of procedures and techniques to solve a certain type of (repetitive) exercises flexibly, accurately and efficiently
- **Strategic competence:** insight, being able to determine strategy and find a good solution
- **Adaptive reasoning:** being able to reflect, check whether you reasoned correctly and make adjustments if necessary
- **Positive disposition:** find mathematics meaningful, useful and worthwhile, coupled with the belief that it pays to work to become more mathematically competent. Experience satisfaction and success when continuing to search for the solution

Kilpatrick et al.

Kilpatrick's model



Understanding concepts, operations and relationships between them
Knowledge

Determine strategy to arrive at a good solution
Insight

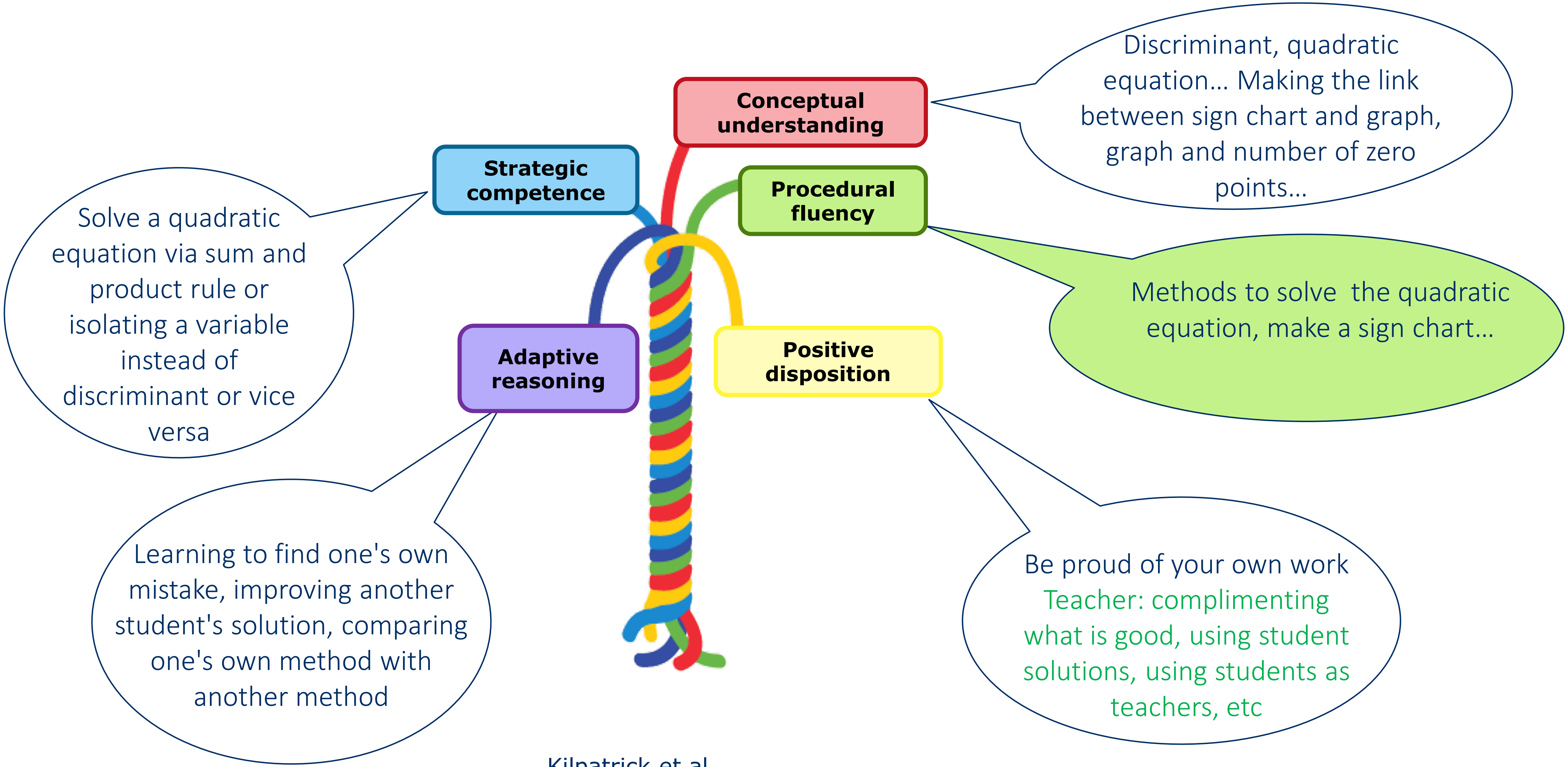
Procedures, methods, step-by-step plans, … deploy flexibly, accurately, efficiently and correctly
Knowledge

Reflection, checking whether you reasoned correctly and making adjustments if necessary
Skills

Positive attitude towards mathematics. It is useful, sensible, effective…
Attitude

Kilpatrick et al.

Link with the case: $f(x) = 2x^2 + 2x - 4 \quad y = -x^2 + 5x$



Kilpatrick et al.

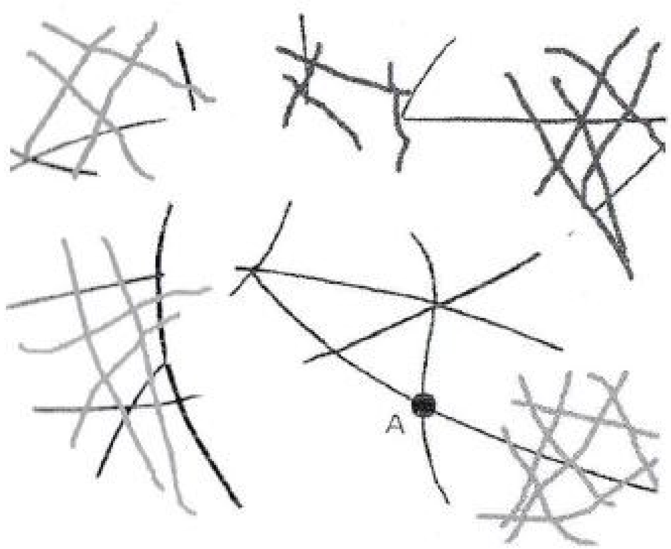
Exercise (5')

- What comes to your mind when you think of linear functions?

- Write down one thing on a post-it.

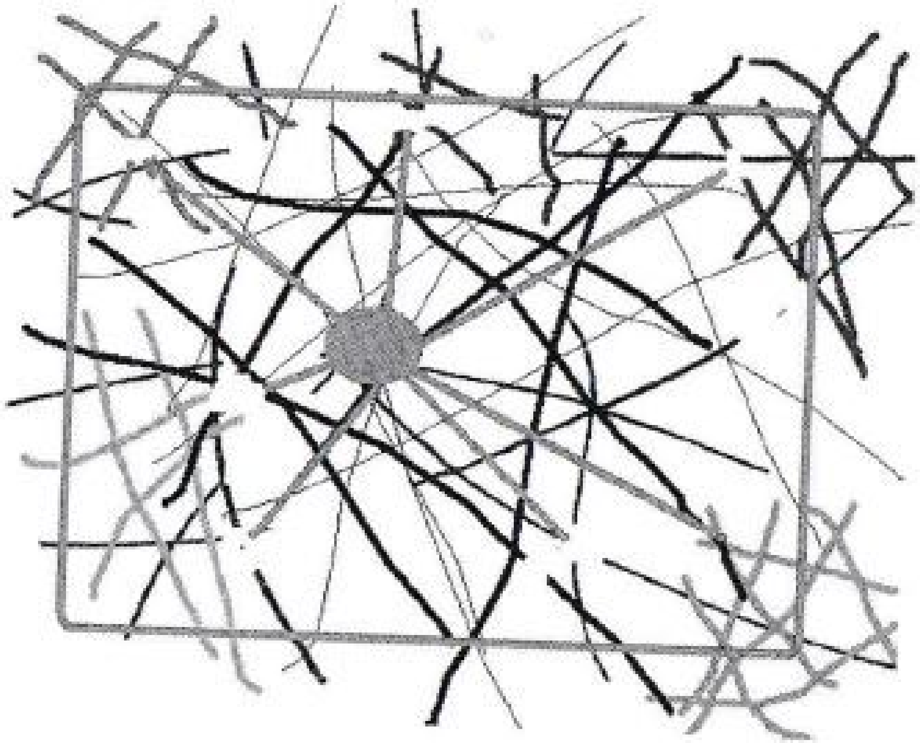
Case: cognitive scheme

Due to the one-sided procedural exercises, students have a **poor cognitive scheme**: fragmented knowledge, few links between course material components (function, second-degree equations, graph, ...)

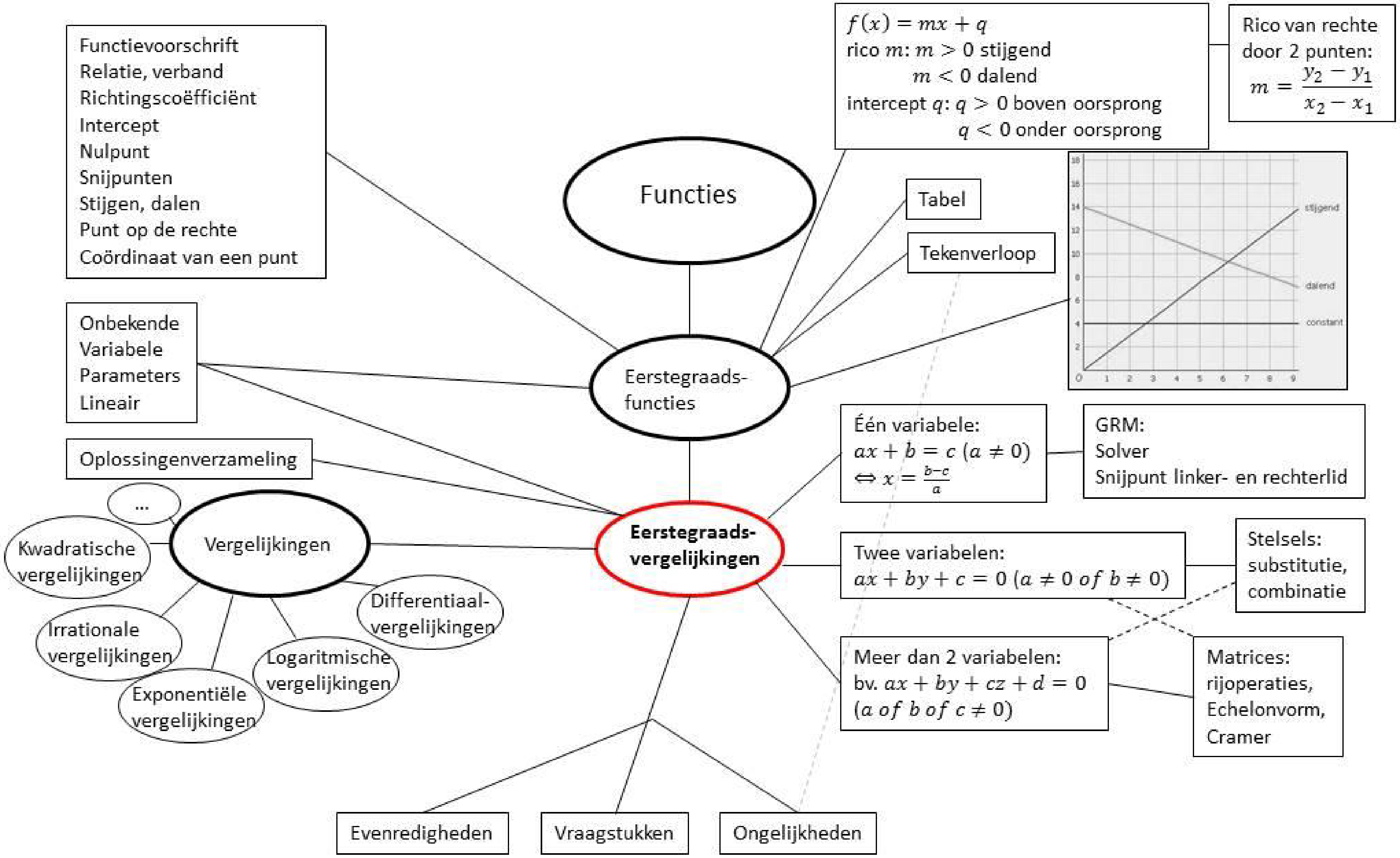


Case: cognitive scheme

Instead, we want students to have a **rich cognitive scheme**, in which links are made between different learning components:



Example of a cognitive scheme



Conclusion

- Our task as a teacher:

Help students
to develop a rich cognitive schema!

- How?
  - Variation in types of exercises (cf. knowledge, insight, etc.).
  - Explicit attention to make connections between learning material components
  - Repeat and connect

Repeat and connect

- With all students AND students themselves
- Brainstorm (via padlet.com)

- Exit-ticket: students write question – with answer → question jar (math of the whole year)

REPEAT MORE – HAVE STUDENTS REPEAT

Questions?

For afternoon programme

- **This room with your own laptop**
- **Universities Computer room with desktop:** s.P.011 (Scribani)

Assignment

- Create a cognitive scheme for the subject

  'definite integral'
  or
  another mathematics subject of your choice

  - 30' individual work
  - 15' supplement your neighbour's scheme
  - Share a photo of your scheme with the others
  - English mathematics textbooks are available

Assignment for the week

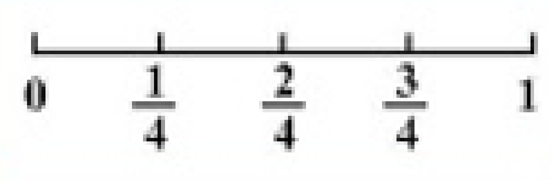
- Based on all the input you receive, prepare a mathematics didactics lesson that you will teach in your own context.
- Gather lesson ideas every day. You will receive flip chart paper, which we will hang around the room. This will serve as input for the 'sharing is caring' session at the end of the day.
- The ideas of the participants will be the topic for discussion during the 'recap of the previous day' session. May be your idea have taken more shape or you might have questions or concerns about it.
- At the end of the week, you will present your work during a brief presentation. We look forward to creative presentations.

Plan B

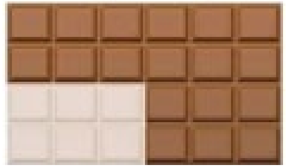
How can we support students in acquiring mathematical proficiency through the didactic lesson structure?

E.g. How would you introduce the concept of fractions?

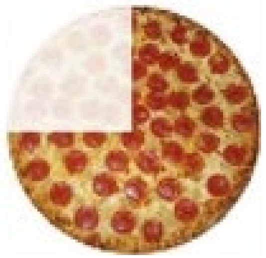
$\frac{a}{b}$



$\frac{3}{4}$

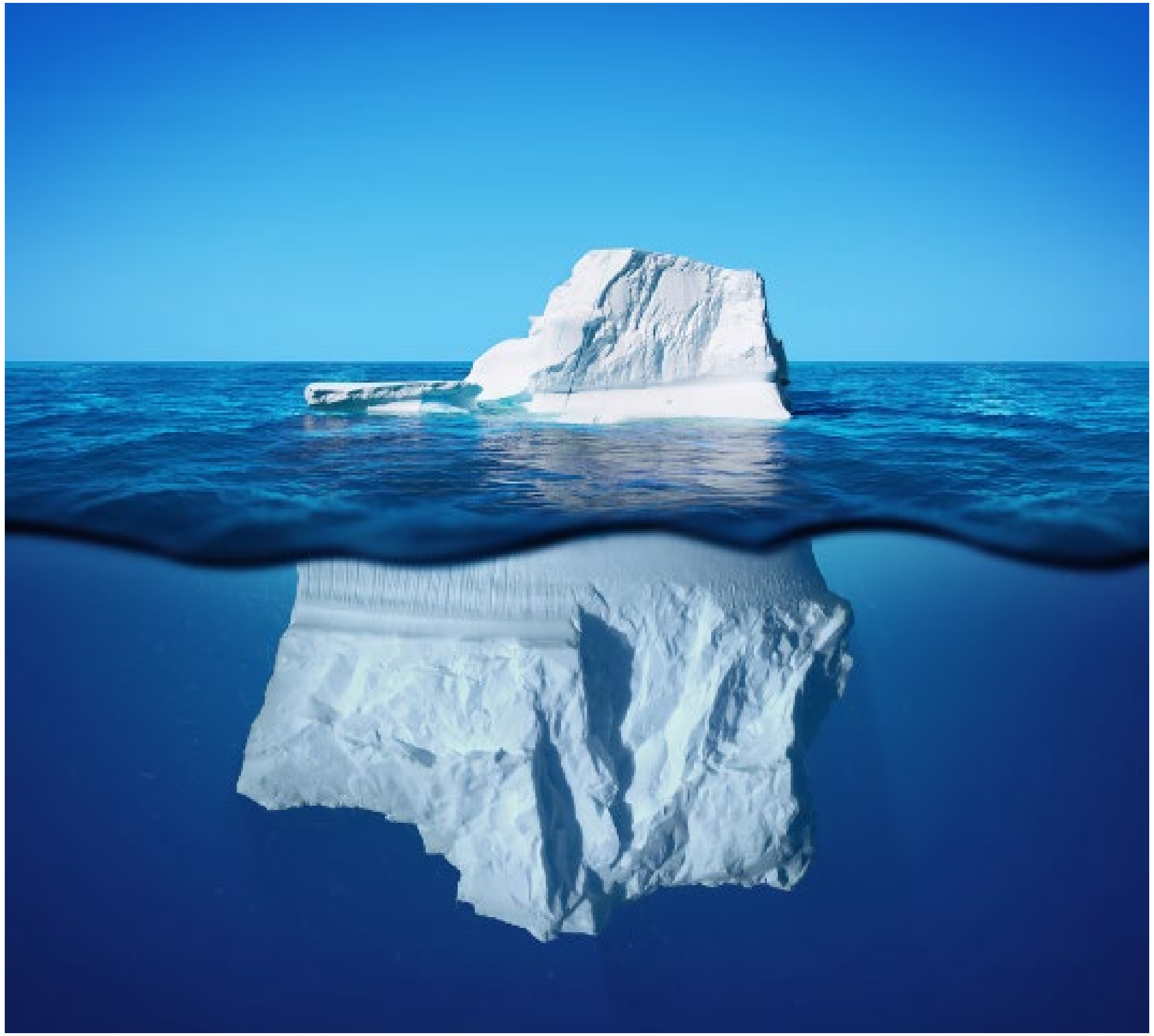


$\frac{1}{2}$ *plus* $\frac{1}{4}$ *pizza*



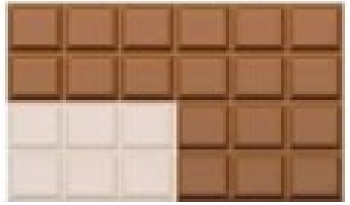
numerator (number of parts we have) $\frac{2}{5}$ denominator (total parts in whole)

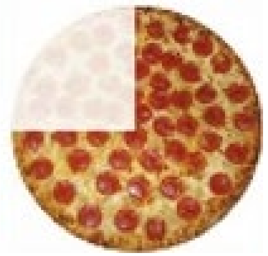
Iceberg metaphor



- Only the tip of an iceberg can be seen and the rest of the iceberg, which is much larger, is underneath the water and cannot be seen.
- Before a student knows and can use the fraction ¾, he/she must first have acquired a lot of other insights.

Iceberg metaphor for fraction ¾ - Step 1

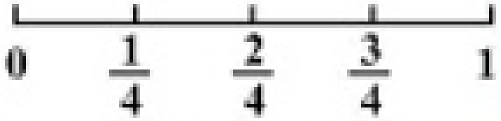




- Start from concrete examples.
- These **informal, context-based examples** are an important link in building rich cognitive schemas.
- They ensure that new knowledge is not abstract and isolated, but is interwoven with what is already known.

Iceberg metaphor for fraction ¾ - Step 2

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$



$$\frac{1}{2}\ plus\ \frac{1}{4}\ pizza$$

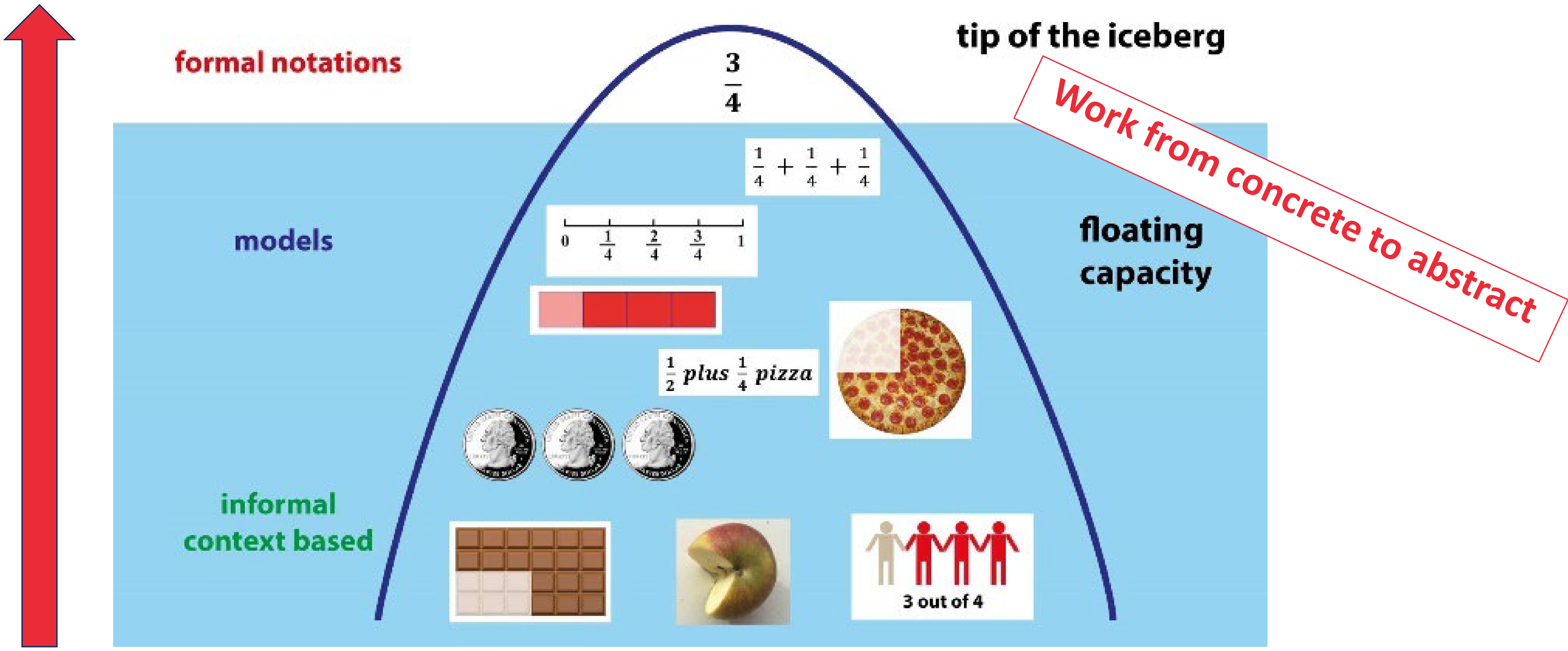
- Continue step by step from concrete to abstract.
- Try using to structure and abstract **didactic models** (e.g. figure, table, sketch, preliminary notation).
- These didactic models represent the essence of the problem.

Iceberg metaphor for fraction ¾ - Step 3

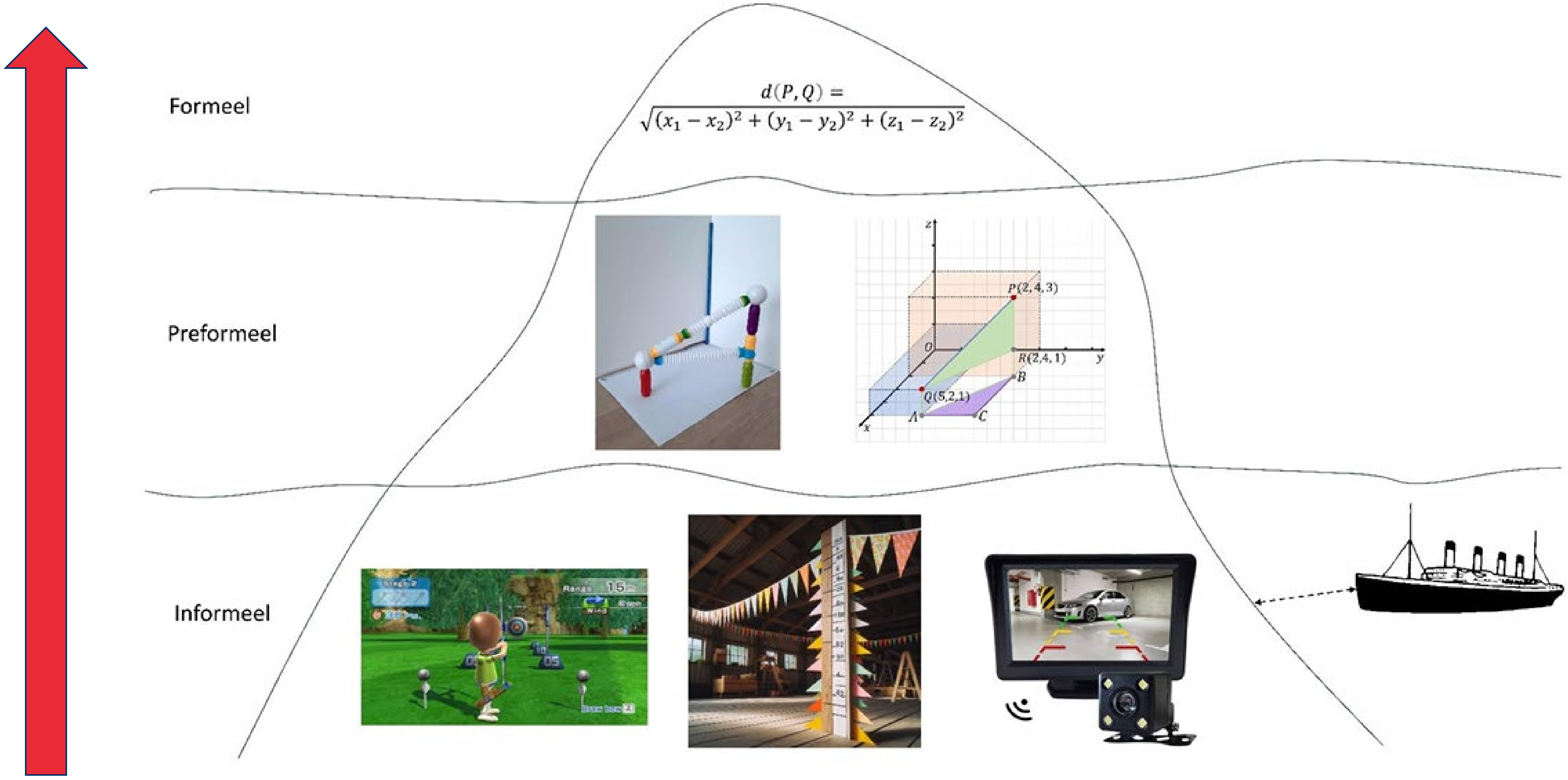
$$\frac{3}{4}$$

- Work towards a **formal notation** based on the concrete examples and didactic models.

Iceberg metaphor for fraction ¾



Iceberg metaphor for the distance formula



Work from concrete to abstract

Hans Freudenthal, mathematician and mathematics didactician from the Netherlands:

*“**No mathematical idea has ever been published in the way it was discovered**. Techniques have been developed and are used, if a problem has been solved, to turn the solution procedure upside down,[…] This is the didactical inversion, which as it happens may be anti-didactical. **Rather than behaving antididactically, one should recognise that the young learner is entitled to recapitulate in a fashion the learning process of mankind.** Not in the trivial manner of an abridged version, but equally we cannot require the new generation to start just at the point where their predecessors left off.”*

(from: H. Freudenthal, Didactical Phenomenology of Mathematical Structures, 1983)

Work from concrete to abstract

Hans Freudenthal, mathematician and mathematics didactician from the Netherlands:

- The order in which a completed piece of mathematics is written down (e.g. proof of a property in an article or in a book) differs from the order in which this piece of mathematics was discovered. **Often the order is, as it were, reversed**.

- The order in a finished mathematical theory is often not the best order to use when learning mathematics. Freudenthal calls this change in sequence (and more broadly: change in organization of the theory) anti-**didactic inversion**.

Flashback

How can we support students in acquiring mathematical proficiency through the didactic lesson structure?

Conclusion

- Divide the lesson into smaller parts ('**lesson sequences**'); each sequence by itself must form a small coherent whole AND the lesson sequences together must also form a large whole.
- Work from **concrete to abstract** (cf. iceberg metaphor).
- Use **didactic inversion** when introducing new learning material. The order in which a completed piece of mathematics is written down is often the reverse of the order in which this piece of mathematics was discovered.